Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter A3

A MODULAR FINITE-ELEMENT MODEL (MODFE) FOR AREAL AND AXISYMMETRIC GROUND-WATER-FLOW PROBLEMS, PART 1: MODEL DESCRIPTION AND USER'S MANUAL

By Lynn J. Torak

Book 6

MODELING TECHNIQUES

U.S. DEPARTMENT OF THE INTERIOR
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PREFACE

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The unit of publication, the chapter, is limited to a narrow field of subject matters. This format allows flexibility in revision and publication as the need arises. Chapters 6A3, 6A4, and 6A5 are on the use of a particular transient finite-element numerical method for two-dimensional, ground-water-flow problems. These Chapters (6A3, 6A4, and 6A5) correspond to reports prepared on the finite-element model given the acronym MODFE and designated as parts 1, 2, and 3, respectively. Part 1 is on "model description and user's manual," part 2 is on "derivation of finite-element equations and comparisons with analytical solutions," and part 3 is on "design philosophy and programming details."

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[1]This manual is a revision of "Measurement of Time of Travel and Dispersion in Streams by Dye Tracing," by E.F. Hubbard, F.A. Kilpatrick, L.A. Martens, and J.F. Wilson, Jr., Book 3, Chapter A9, published in 1982.

[2]Spanish translation also available.

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[1]This manual is a revision of TWRI 5–A3, "Methods of Analysis of Organic Substances in Water," by Donald F. Goerlitz and Eugene Brown, published in 1972.

[2]This manual supersedes TWRI 5–A4, "Methods for collection and analysis of aquatic biological and microbiological samples," edited by *P.E. Greeson and others, published in 1977.*

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A MODular Finite-Element Model (MODFE) for Areal and Axisymmetric Ground-Water-Flow Problems Part 1: Model Description and User's Manual

By Lynn J. Torak

Abstract

A *MOD*ular, *F*inite-*E*lement digital-computer program (MODFE) was developed to simulate steady or unsteady-state, two-dimensional or axisymmetric ground-water flow. Geometric- and hydrologic-aquifer characteristics in two spatial dimensions are represented by triangular finite elements and linear basis functions; one-dimensional finite elements and linear basis functions represent time. Finite-element matrix equations are solved by the direct symmetric-Doolittle method or the iterative modified, incomplete-Cholesky, conjugate-gradient method. Physical processes that can be represented by the model include (1) confined flow, unconfined flow (using the Dupuit approximation), or a combination of both; (2) leakage through either rigid or elastic confining beds; (3) specified recharge or discharge at points, along lines, and over areas; (4) flow across specified-flow, specified-head, or head-dependent boundaries; (5) decrease of aquifer thickness to zero under extreme water-table decline and increase of aquifer thickness from zero as the water table rises; and (6) head-dependent fluxes from springs, drainage wells, leakage across riverbeds or confining beds combined with aquifer dewatering, and evapotranspiration.

The report describes procedures for applying MODFE to ground-water-flow problems, simulation capabilities, and data preparation. Guidelines for designing the finite-element mesh and for node numbering and determining band widths are given. Tables are given that reference simulation capabilities to specific versions of MODFE. Examples of data input and model output for different versions of MODFE are provided.

Introduction

This is the first report of a three-part series of reports that documents MODFE, a MODular, Finite-Element, digital-computer program. This report is intended to be a user's manual that describes applications of MODFE for simulating the physical processes associated with two-dimensional and axisymmetric ground-water flow. The development of matrix equations that are solved by MODFE is given in a companion report, Part 2 (Cooley, 1992). Similarly, details of the modular program design are given in Part 3 (Torak, 1993). Simulation capabilities of MODFE include:

- transient or steady-state conditions,
- nonhomogeneous and anisotropic flow where directions of anisotropy change within the model region,
- vertical leakage from a semiconfining layer that contains laterally nonhomogeneous properties and elastic storage effects,
- point and areally distributed sources and sinks,
- specified-head (Dirichlet), specified-flow (Neumann), and head-dependent (Cauchy-type) boundary conditions,
- vertical cross sections,
- axisymmetric-cylindrical flow,
- confined and unconfined (water-table) conditions,
- partial drying and resaturation of a water-table aquifer,
- conversion between confined- and unconfined-aquifer conditions,
- nonlinear-leakage functions (for simulating line, point, or areally distributed sources and sinks),
- changing stresses and boundary conditions on a stress-period basis, time-step basis, or both, and
- zoned input of hydraulic properties and boundary conditions.

Elements of the simulation capabilities listed above are described with regard to their representation of hydrologic characteristics of ground-water-flow problems and their implementation in MODFE. Examples are given that show alternate formulations for representing the same hydrologic characteristic, such as leakage to or from a river, and the circumstances under which each form would apply during simulation.

A discussion of design considerations for the finite-element mesh is given to provide important background information to the model user about creating a mesh with the appropriate subdivision (discretization) where needed within the region to be simulated. Methods for recognizing and minimizing errors in the computed solution that are related to the discretization scheme also are provided. Generalized rules are given as guidelines to proper mesh construction for most applications of MODFE.

Several methods of improving the computational efficiency and ease of data input are discussed in this report. These include selecting either a direct or an iterative method to solve the finite-element equations, numbering nodes in the finite-element mesh to ensure efficient use of computer storage and of the direct-solution method, and preparing hydrologic information for data input by zone. Example simulations are used to demonstrate data input and program output, and input instructions are given.

Background

Descriptions of the numerical representation of physical processes and hydrologic features contained in this and the companion reports have evolved over the past 10 years from material presented by the authors in the course "Finite-Element Modeling of Ground-Water Flow," held at the U.S. Geological Survey National Training Center in Denver, Colorado. This report formalizes the course material, which has been revised to incorporate comments and suggestions from attendees of the courses.

Purpose and Scope

This report is a guide to using MODFE for simulating two-dimensional, ground-water-flow problems of varying complexity. Concise descriptions are given for representing hydrologic characteristics of ground-water-flow problems numerically and by the finite-element method used in MODFE. Discussions of simulation capabilities emphasize how the model user would convert real-world, hydrologic conditions into a numerical representation for solution. Details concerning computational aspects of MODFE are not included in this report. In like manner, the model equations also are not developed. These aspects of MODFE are given proper emphasis in companion reports by Cooley (1992) and Torak (1993), so that this report can focus on the hydrologic aspects of applying MODFE to solve ground-water-flow problems. Hence, a user can begin to simulate aquifer problems with MODFE after only a short investment in time spent reading this report.

Governing Equation

MODFE solves the two-dimensional, unsteady-state equation of ground-water flow given by equation (1) in Cooley (1992), for hydraulic head h, subject to the boundary conditions expressed by equations (2) through (5) in Cooley (1992). This equation is restated as

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}+T_{xy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial y}\left(T_{yx}\frac{\partial h}{\partial y}+T_{yy}\frac{\partial h}{\partial y}\right)+R(H-h)+W+P=S\frac{\partial h}{\partial t} \quad (1)$$

where

(x,y)= Cartesian coordinate directions [length],

t= time [time],

h(x,y,t)= hydraulic head in the aquifer [length],

H(x,y,t)= hydraulic head in the source layer [length],

$\begin{bmatrix} T_{xx}(x,y,h,t) & T_{xy}(x,y,h,t) \\ T_{yx}(x,y,h,t) & T_{xy}(x,y,h,t) \end{bmatrix}$ = symmetric transmissivity tensor written in matrix form [length^2/time]

R(x,y,t)= hydraulic conductance (vertical hydraulic conductivity of a confining bed divided by its thickness) [time^{-1}],

S(x,y,t)= storage coefficient [0],

W(x,y,t)= unit areal recharge or discharge rate (positive for recharge) [length/time], and

$P(x,y,h,t)=\sum_{j-1}^{p}\delta(x-\alpha_j)\delta(y-b_j)Q_j=$ Dirac-delta designation for p point sources or sinks, each of strength Q_j [length/time] and located at (a_j,b_j). Q_j is positive for injection.

Both confined (linear) and water-table (nonlinear) conditions are simulated by MODFE and are represented by equation (1). For confined ground-water flow having linear boundary conditions, terms in equation (1) that multiply either hydraulic head, h(x,y,t), or derivatives of head, or terms that represent boundary flows, are not functionally dependent on head and are constant in time. An example is ground-water flow in a confined aquifer having linear boundary conditions, where neither the transmissivity nor storage coefficient are functions of head.

When applied to nonlinear ground-water flow, equation (1) contains terms that are functionally dependent on hydraulic head. For example, in a water-table (unconfined) aquifer, transmissivity is a function of hydraulic conductivity, K, and changing aquifer thickness, b, which is a function of changing hydraulic head. That is, $b = h - z_b$, where z_b is the altitude of the aquifer bottom, and $T = K(h - z_b)$, (see equation (65) of Cooley, 1992). Another example of nonlinear conditions occurs when aquifers convert from confined to unconfined (or from unconfined to confined) conditions during the simulation, where the storage coefficient and transmissivity change with time depending on the value of hydraulic head relative to the base of the overlying confining bed.

Terms accounting for steady vertical leakage, R(H − h) in equation (1), or Cauchy-type boundaries, $\alpha(H_B - h)$ in equation (4), (Cooley, 1992), will cause equation (1) to be nonlinear if R, α, or the head differences change as a function of aquifer head. Typical applications of these nonlinear-leakage functions are given in subsequent sections of the user's manual and involve simulation of rivers, springs or drainage-well discharge, evapotranspiration, and vertical leakage from an overlying confining bed when the aquifer converts from confined to unconfined (or from unconfined to confined) conditions.

Procedure for Applying the MODular Finite-Element Model (MODFE) to Ground-Water-Flow Problems

The procedure for applying MODFE to two-dimensional ground-water-flow problems begins by constructing a finite-element mesh that will accurately represent hydrologic factors such as hydraulic head, aquifer geometry, and boundary conditions within the region to be simulated. The following section provides guidelines for mesh construction, and gives examples of how these hydrologic aspects can be represented by subdivision, or discretized, with triangular elements, element sides, and (or) element intersections (nodes) in a finite-element mesh.

After a suitable finite-element mesh is constructed, the next step is to number the nodes in a manner that minimizes computer storage and execution time; this increases the computational efficiency of MODFE. Techniques are discussed for minimizing the maximum difference between node numbers in any element (termed the matrix bandwidth) and for minimizing the maximum number of element connections to any node (termed the condensed semibandwidth), two factors related to mesh design and node numbering that affect computational efficiency. Examples are given that show node-numbering schemes and their effects on computer storage, and how to eliminate excessive element connections to any node by redesigning parts of the mesh. Although these discussions follow the section on mesh design, they provide valuable information that a user should consider during construction of a finite-element mesh.

The next step in applying MODFE to solve a ground-water-flow problem is to select a solution method to the finite-element matrix equations that are formed by the program. Information is provided about the direct (symmetric-Doolittle) method of triangular decomposition and about the iterative, modified Cholesky, conjugate-gradient method so that the user can decide which method will better suit the aquifer problem to be solved. Like the previous considerations for mesh design and node numbering, selection of either solution method is a matter of computational efficiency rather than numerical accuracy.

Design Considerations for the Finite-Element Mesh

This section describes how to design a finite-element mesh to account for the pertinent hydrologic factors that are present in ground-water-flow problems that can be solved by using MODFE. The accurate representation of hydrologic factors that affect ground-water flow in an aquifer depends partly on accurate approximations to the ground-water-flow equations by the finite-element method, as discussed in Cooley (1992), and partly on the design of the finite-element mesh. Factors such as locations of flow boundaries and wells, aquifer geometry, distribution of hydrologic properties, and shape of the anticipated potentiometric surface, all influence mesh design. These factors are discussed relative to representing an aquifer region with triangular elements, element sides, and nodes, which together comprise a finite-element mesh.

Parts of this section contain descriptions of concepts of the finite-element method that were developed in Cooley (1992) and that influence the design of a finite-element mesh. Descriptions are given about the manner in which the finite-element method allows hydraulic head and hydrologic properties to vary within the area to be simulated, and how these variations can be represented accurately by the mesh design. A discussion is presented about errors in the computed heads over space and time that are caused by a poorly designed mesh (termed discretization errors), that is, a mesh that does not adequately account for these concepts, and guidelines are given so that a user can test for and minimize the effects of these errors.

Concepts of the Finite-Element Method

Before discussing how a finite-element mesh can be constructed to represent the hydrologic factors that influence ground-water flow in an aquifer, some basic concepts about the finite-element method used in the development of MODFE are described as they relate to mesh design. As stated in Cooley (1992), a basic concept of the finite-element method is that a complex flow region can be subdivided into a network of subregions, called elements, each having a simple shape. The element shapes used by MODFE are triangles, which can approximate curved boundaries of the flow region by careful placement of element sides. Elements are constructed over the region so that the sides of one element coincide completely with the sides of adjacent elements. Elements are joined along common sides and at vertices, which are called nodes, and nodes are located only at element vertices (fig. 1*A*). Hydraulic head is computed at the node points by MODFE. The network of triangles over the flow region is called a finite-element mesh.

The concept of variation in hydraulic head within an element is discussed briefly as it relates to mesh design. Hydraulic head is approximated at any point, (x,y), within an element, e, by a simple algebraic equation, or function, for a plane, $\hat{h} = A^e + B^e x + C^e y$ (equation (6) in Cooley, 1992). Values for the function, $\hat{h}$, are defined at the node (fig. 1*B*). This allows coefficients A^e, B^e, and C^e to be determined from nodal values of $\hat{h}$. The relations of A^e, B^e, and C^e to $\hat{h}$ permits $\hat{h}$ to vary linearly within an element, and solution of h within an element is known, given the values of $\hat{h}$ at the nodes. (See development of equations (6) through (10) in Cooley (1992), for details.)

Each element in the finite-element mesh represents a plane of the function $\hat{h}$, which represents hydraulic head. The orientation of the plane in space depends on the values of $\hat{h}$ at the nodes (fig. 1*B*). On an element side, $\hat{h}$ is a linear function of head at the nodes that define the side; the head at the third node in the element is not used to define $\hat{h}$ along this side. Thus, sides of adjacent elements have the same orientation in space so that the mesh forms a network of piecewise continuous planes of $\hat{h}$ within the aquifer region (fig. 1*C*).

Shape of the Anticipated Potentiometric Surface

An important consideration in the design of the finite-element mesh is the shape of the anticipated solution of hydraulic head, or, of the potentiometric surface, within the aquifer region. The network of triangular planes that comprises a finite-element mesh has the versatility to represent changes in hydraulic heads and in gradients that occur within the aquifer region during simulation, provided that these changes are anticipated and are accommodated by the mesh design. In designing a finite-element mesh, the user should identify any anticipated stresses on the flow system and attempt to understand how these stresses will affect the simulated potentiometric surface. For example, initiating or discontinuing well pumpage, or changing existing well-pumping rates or boundary flows, can cause changes in hydraulic gradients over short distances within the aquifer region. Lateral discontinuities in aquifer properties, caused by abrupt changes in aquifer thickness or lithology, can cause hydraulic gradients to change as the discontinuities are crossed.

For a finite-element mesh to represent a potentiometric surface accurately, the network of triangular planes should be able to approximate curved surfaces well. As discussed in the previous section, each element is a plane that represents part of the potentiometric surface. The orientation of each plane is defined by the values of head at the three nodes describing the element (fig. 1*B*,*C*). On a highly curved potentiometric surface, such as near a pumped well or wells, many elements of small area are required to represent the changing orientation (hydraulic gradients) of the surface (fig. 2*A*). Conversely, where the potentiometric surface is nearly flat and contains only slight changes in hydraulic gradient, such as in an undeveloped, relatively homogeneous aquifer, elements of larger area than those used near a pumped well would be sufficient for representing the surface (fig. 2*B*). However, if the effects of a well field or similar stress is to be simulated on the undeveloped aquifer, then small elements should be used in regions where the anticipated stress is to be applied so that changing hydraulic gradients can be represented adequately by the mesh during simulation of the anticipated stress.

The steepness of hydraulic gradients in the anticipated solution is not as important a design consideration as the change in gradients with distance. Hydraulic gradients that are constant or that do not vary appreciably within the aquifer region, regardless of slope, indicate a relatively noncurving potentiometric surface that can be approximated accurately by using a few large elements. However, where a transition between steep and gentle gradients occurs, or where a transition is anticipated, a curved potentiometric surface can be created. These areas might require subdivision by using many small elements (fig. 3).

Aquifer Geometry and Hydrologic Boundaries

Curved or irregularly shaped aquifer geometry and hydrologic boundaries influence the potentiometric surface within an aquifer region, and therefore, influence the design of the finite-element mesh. Locations of these features can be approximated nearly exactly in a mesh by using element sides, as shown in figure 4. Element sides that define the irregular geometry of aquifer-region boundaries can be used to represent specified-head (Dirichlet), specified-flow (Neumann), or mixed (Cauchy-type) conditions. The location of zone boundaries for hydraulic properties can be represented accurately with element sides (fig. 4).

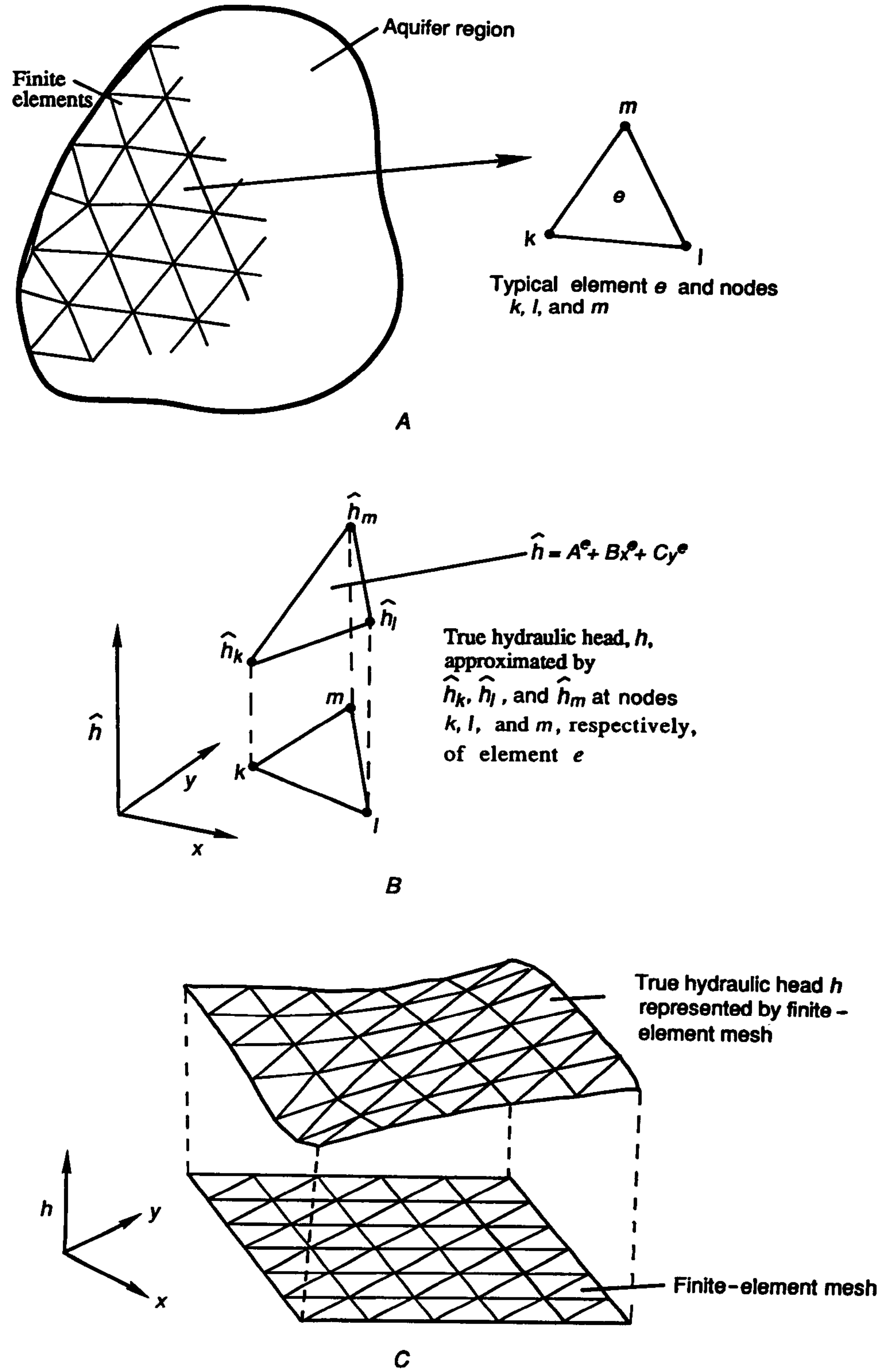


Figure 1.—Diagrams showing (*A*) aquifer region partially subdivided by finite elements and typical element *e*; (*B*) finite-element representation of hydraulic head $\hat{h}$; and (*C*) finite-element mesh configuration for approximating true hydraulic head *h*.

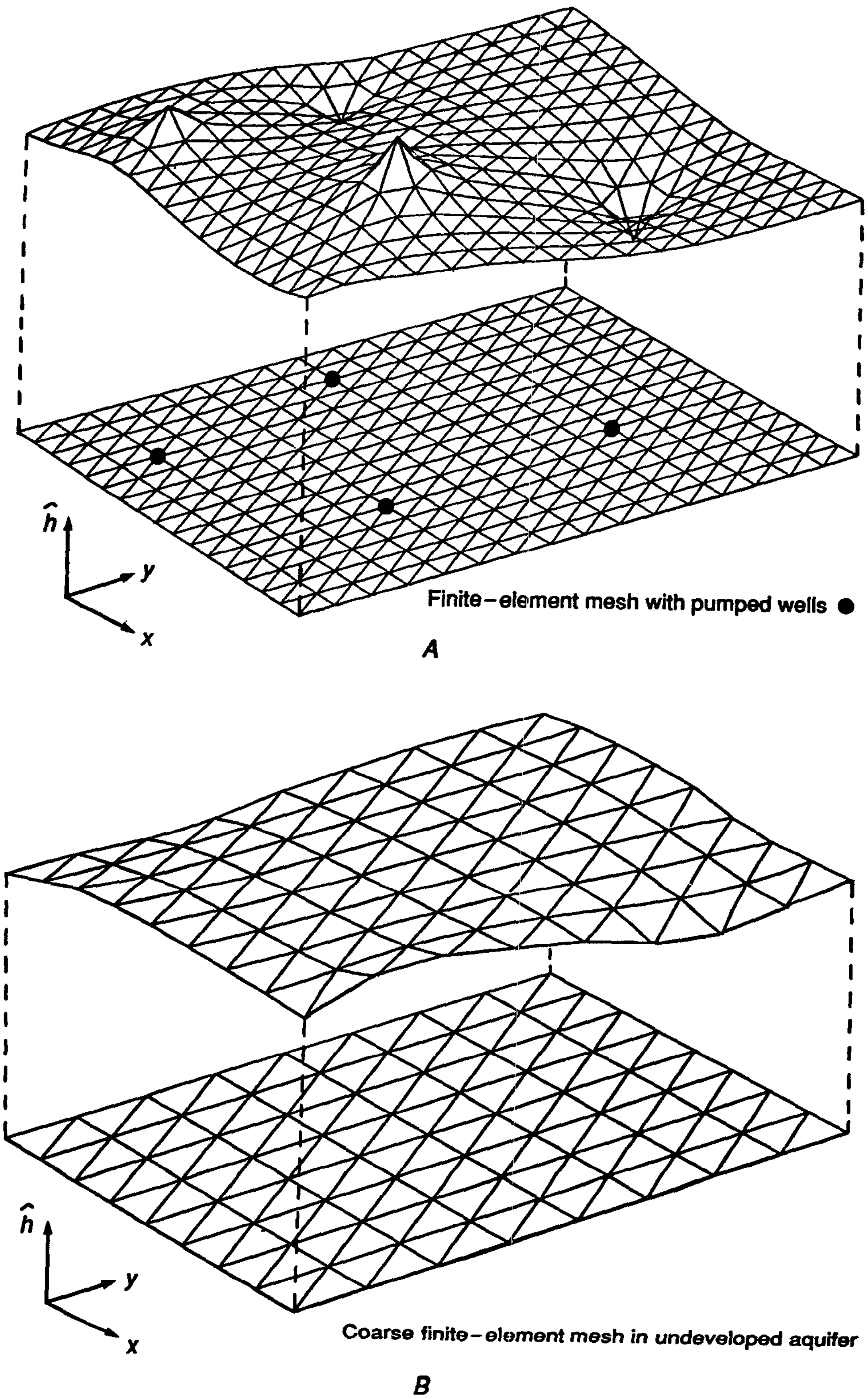


Figure 2.—Representation of potentiometric surface by finite elements (*A*) near pumped wells; and (*B*) in an undeveloped aquifer.

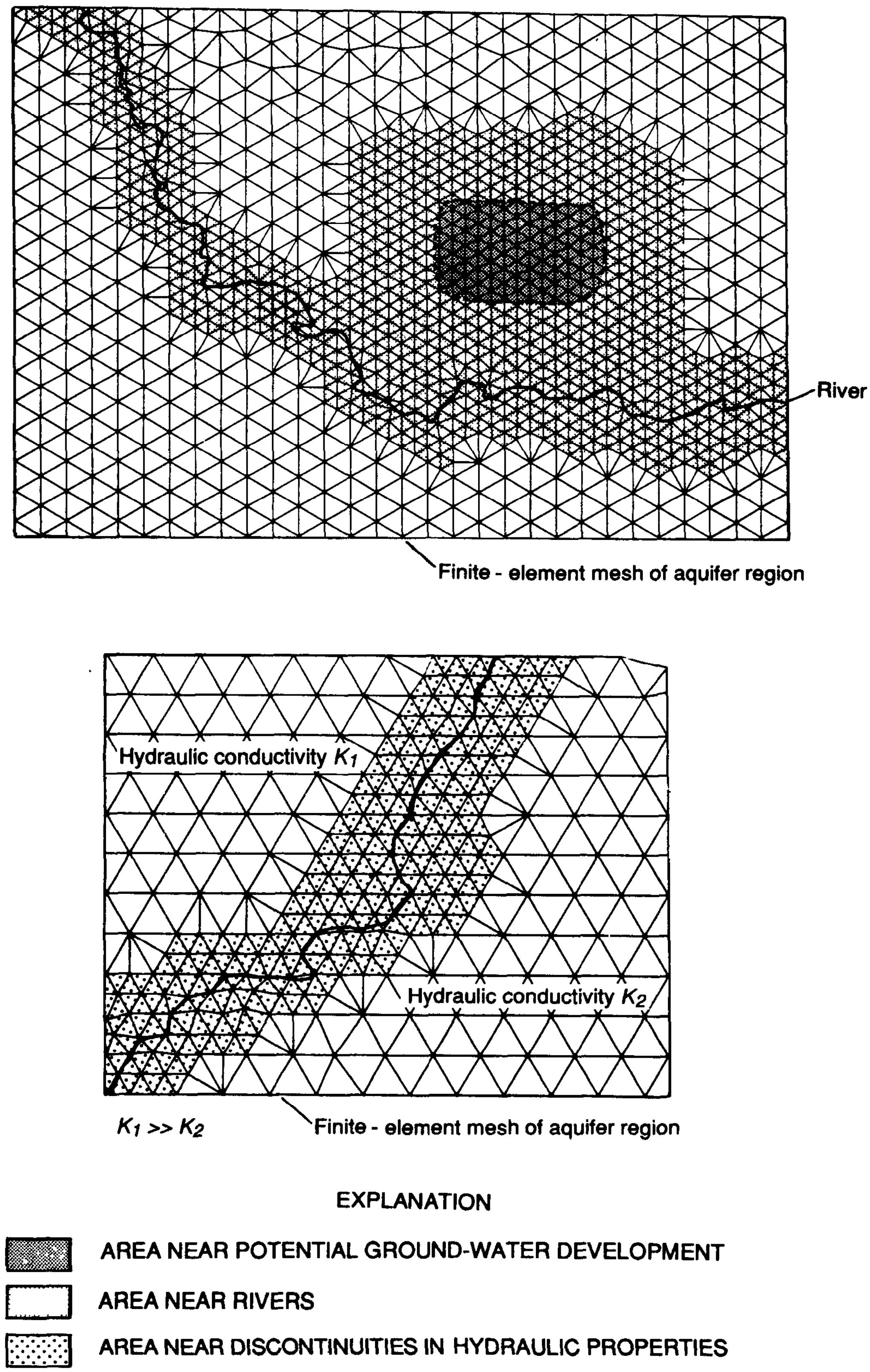


Figure 3.—Examples of additional subdivision by finite elements in areas of changing hydraulic gradient with distance.

As stated in the previous section, changes in hydraulic properties represent discontinuities in the ground-water-flow system that may cause changes in hydraulic gradients within the aquifer region. Hence, accurate location of hydraulic-property zones is necessary to simulate ground-water flow accurately. Rivers also can be represented by element sides in the same manner as hydraulic-property zones. Impermeable (no-flow) boundaries within the aquifer region can be simulated as either a "hole" in the finite-element mesh (fig. 4), or as a zone of elements that contain hydraulic properties of zero. (A discussion of hydraulic-property zones is given in a later section.)

The irregularly shaped and curved contacts between hydrologic units in cross section can be represented easily by element sides. The flexibility in mesh design is particularly useful for representing contacts between units that are folded or arcuate shaped, or where hydrologic boundaries create irregular aquifer geometry (fig 5*A*). Aquifer "pinchouts" or facies changes are represented by positioning element sides along the contacts between units (fig. 5*B*). Combining this aspect of mesh design with the capability to change directions of anisotropy within the aquifer region enables simulation of a wide range of aquifer problems in cross section by using MODFE. (See section "Cross Sections.")

Points of Known Hydraulic Head and Stress

Locations within the aquifer region where stresses and hydraulic heads are known can affect the design of the finite-element mesh. Points of known hydraulic head or stress, such as observation or pumped wells, springs, and drains can be positioned exactly in the finite-element mesh by placing nodes at their locations (fig. 2*A*). The mesh is then constructed with these nodes used as vertices for some of the triangular elements. Other design considerations for the finite-element mesh require that additional nodes and elements be used in order to obtain an accurate approximation of the true potentiometric surface.

Comparison of computed heads with measured values can be made easily from the point (nodal) solutions of hydraulic head that are provided by MODFE. Because the computed head at a node approximates a point on the true potentiometric surface, the computed values can be compared directly with measured water levels (or an analytical solution, as in Cooley, 1992), provided the nodes are located at the points of observation. However, if the nodal locations and the measurements do not coincide, then the expression for the approximate solution of head, $\hat{h}$, given by equation (6) of Cooley (1992), can be used to compute hydraulic head at any (x,y) location within an element. This equation uses nodal coordinates and values of head and coordinates of the measurement to compute the head at the specific location.

General Construction Rules

Some general rules are given as a guide to ensure proper construction of the finite-element mesh. Several design considerations have been mentioned in previous sections, and many different triangular-element shapes are permitted by the finite-element method. However, not all element shapes are acceptable for representing hydrologic conditions in an aquifer region. Adherence to the following rules will enable construction of a mesh that addresses the design considerations and has the ability to provide a numerical solution that closely approximates observed conditions. Some construction rules may not require strict adherence, and in some cases, might be inappropriate or inapplicable given the nature of the aquifer problem. Other rules, if violated, may lead to errors in the numerical solution, termed discretization errors. These errors are described briefly after the associated rule, and a reference is given for the interested reader who wishes to pursue a more detailed discussion. Identification of discretization errors and methods for evaluating and minimizing them are discussed in the following section.

— Subdivide the aquifer region as much as possible by using a regular triangular mesh consisting of uniformly sized equilateral triangles (fig. 4). However, other design considerations within the aquifer region, such as discontinuities in hydraulic properties, aquifer geometry, or hydrologic boundaries, may take precedence over strict adherence to this construction rule. Equilateral triangles provide the most accurate representation of hydrologic conditions by minimizing the error associated with approximating hydraulic head with the finite-element method. For a regular triangular mesh, the error of approximation is proportional to the square of the length of an element side. For a mesh consisting of irregular element shapes, the approximation error is proportional to the square of the maximum length of the element sides (Strang and Fix, 1973, chs. 1 and 2).

— Avoid construction of thin, needle-like elements (fig. 6*A*), as these "degenerate" triangles can either affect the numerical stability of the finite-element method (Strang and Fix, 1973, ch. 3), or introduce errors into the approximation of hydraulic head within an element. Although no limitations on the thinness of an element was found in the literature, Strang and Fix (1973, p. 139) indicated that angles as small as 22.5 degrees did not pose a problem with the finite-element method. My experience with mesh design indicates that angles between 30 and 90 degrees are sufficient for subdividing an aquifer region with finite elements.

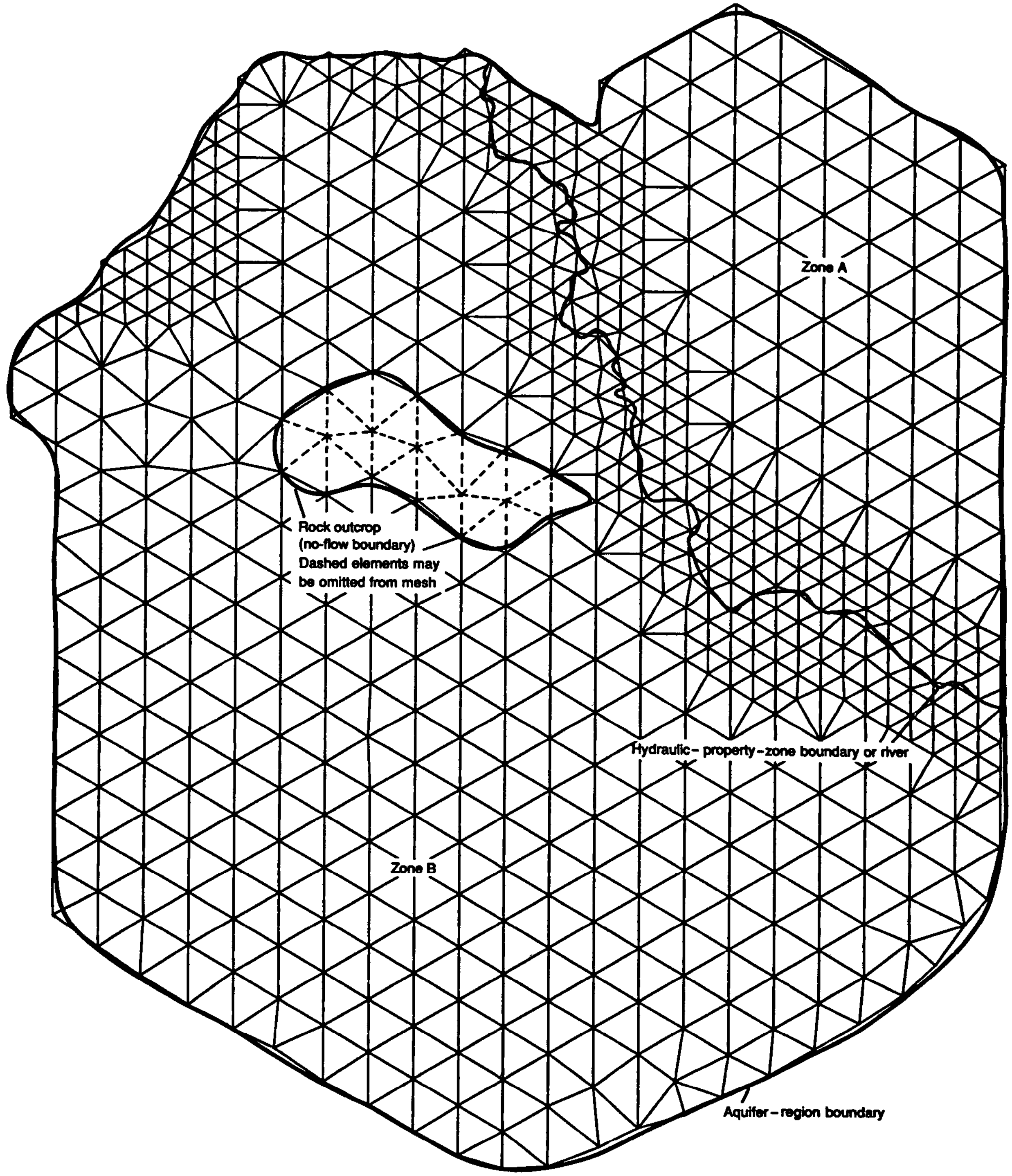


Figure 4.—Approximation by finite elements of curved aquifer geometry and hydraulic-property boundaries.

Smaller angles could be used for specialized problems, such as axisymmetric flow to a well. (See section "Comparisons of Numerical Results with Analytical Solutions," in Cooley, 1992.)

– The sum of two angles created by a side that is common to two elements cannot exceed 180 degrees (fig. 6*B*). Violation of this construction rule results in at least one obtuse angle being created by the side that is shared by the two elements. The existence of obtuse angles in the mesh can be determined by inspection, if the mesh is small, or by a computer program that uses the nodal coordinates as input.

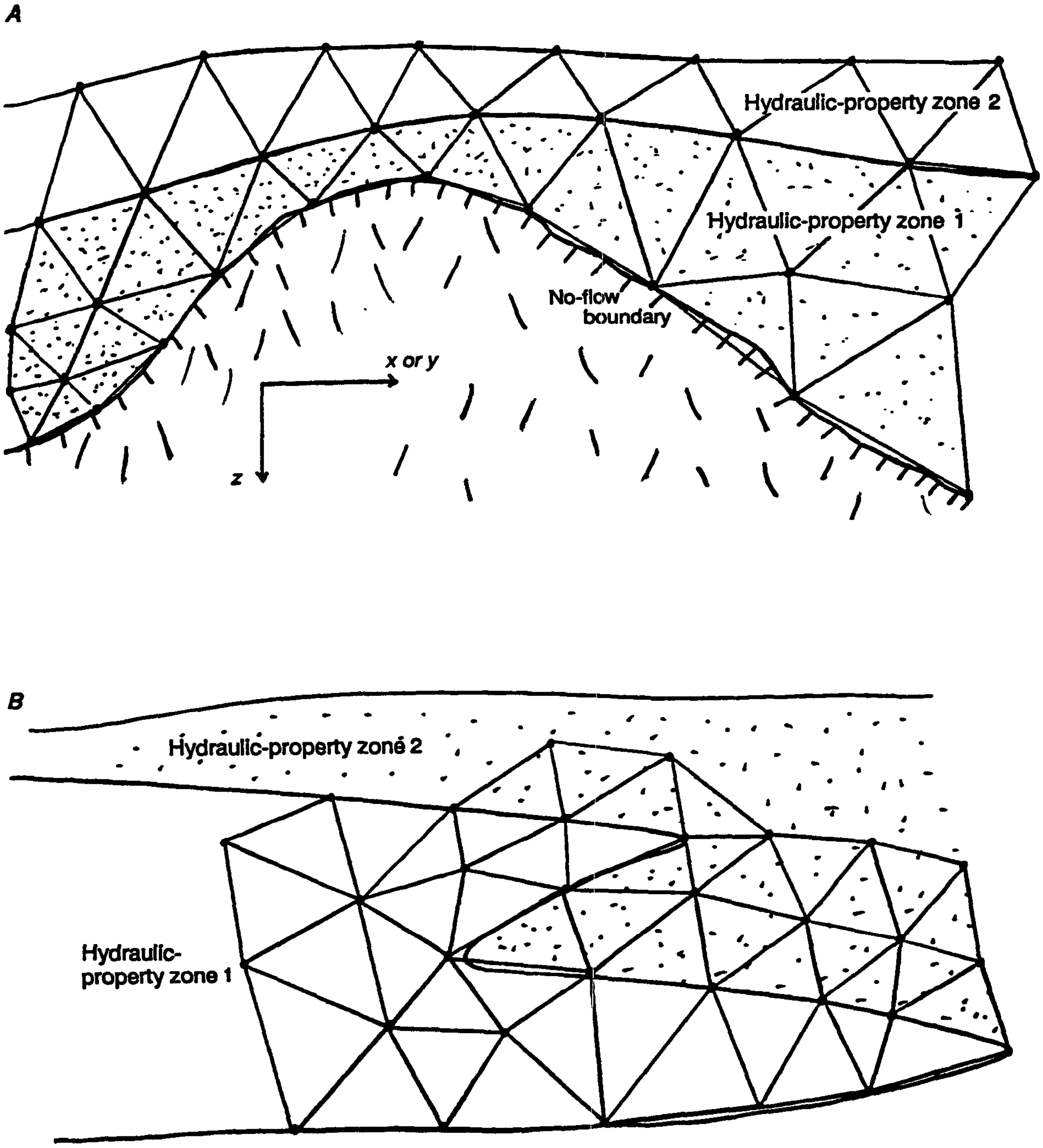


Figure 5.—Representation by finite elements of hydraulic-property zones in cross section (*A*) near arcuate boundaries; and (*B*) along facies changes.

Adherence to this rule ensures diagonal dominance of transmissivity (or hydraulic conductivity) terms in the coefficient matrix G of equation (49) in Cooley (1992). That is, all off-diagonal transmissivity (or hydraulic conductivity) terms in the coefficient matrix are negative, and all main diagonals are positive. This guarantees a non-negative inverse matrix of G, a desirable condition for numerical solution. Because the inverse matrix and the coordinate functions N of equation (9) in Cooley (1992) are non-negative, the finite-element representation of hydraulic head obeys the same physical laws as the true hydraulic head. That is, heads will decrease (increase) when a negative (positive) stress is applied. (See Strang and Fix, 1973, p. 78).

— Incorporate as much aquifer-problem geometry into the finite-element mesh as possible. Curved hydro-

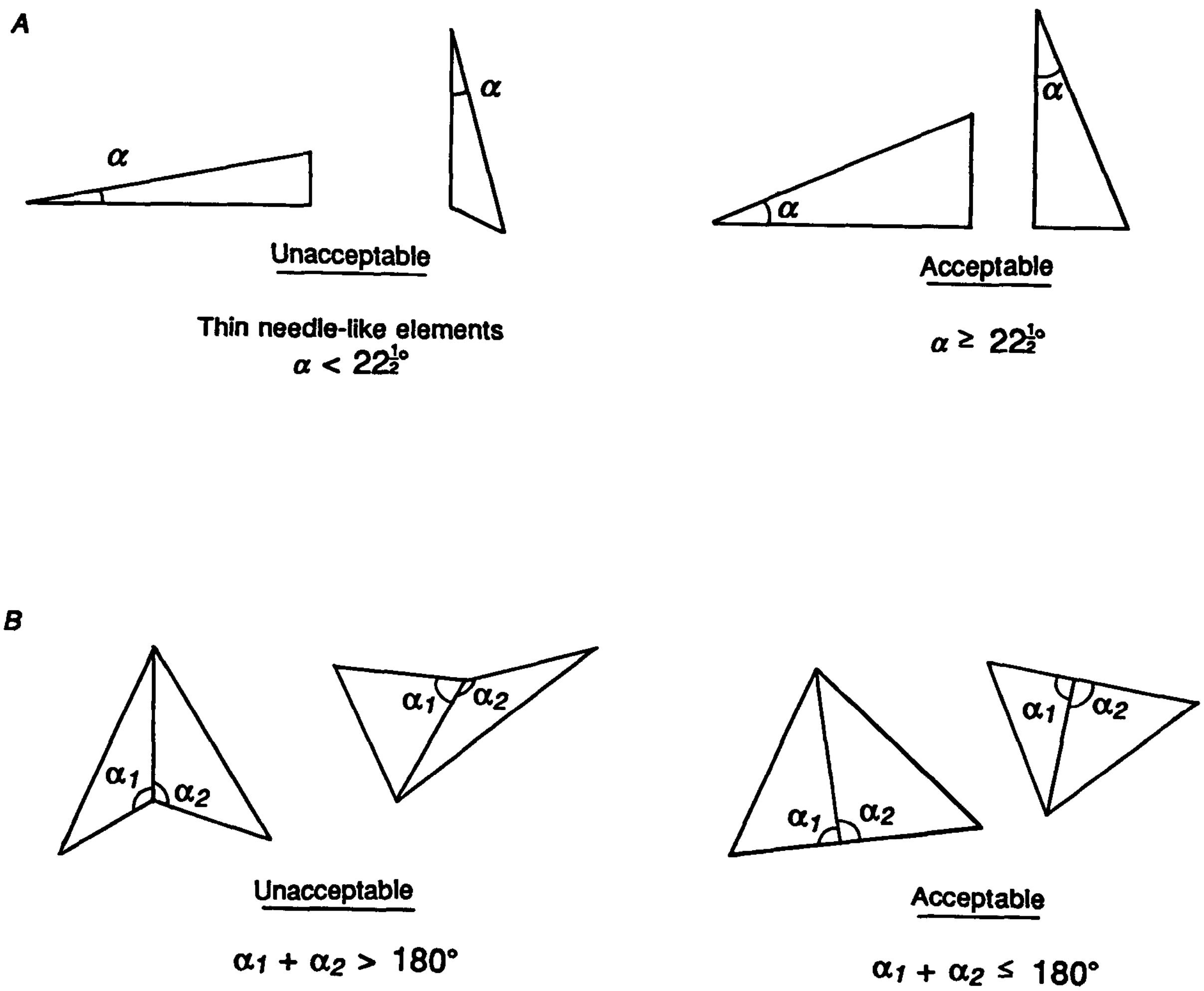


Figure 6.—Acceptable and unacceptable element shapes.

logic boundaries are represented easily by locating nodes on the boundaries (fig. 4). Symmetry in the anticipated solution of hydraulic head requires a corresponding symmetry in the design of the mesh, such as around cones of depression near pumped wells or other stresses (fig. 2).

— Locate points of observation and hydrologic features that are pertinent to an aquifer problem as exactly as possible by using nodes or element sides. Water-level measurements, stresses, and boundary conditions can be represented within the finite-element mesh either as nodes or element sides, allowing for direct comparison of measured values for hydraulic head and flux with the computed solution. However, if nodes cannot represent the location of a water-level measurement exactly, then the value at that location can be determined from values at the surrounding nodes, as described in the section "Points of Known Hydraulic Head and Stress."

— Nodes can be positioned only at vertices of triangular elements.

— Design the mesh so that the number of element sides that connect to any node is fairly uniform throughout the mesh. This construction rule is related more to effective use of computer storage and computation time by MODFE than to the ability of the mesh to represent true hydrologic conditions. This design criterion influences the value of the condensed-matrix bandwidth, which is discussed in the following section.

For most aquifer problems in areal dimensions, a regular triangular mesh can be either drawn by hand or generated by computer to cover the extent of the aquifer region of interest (fig. 4). If additional discretization is needed within the simulated region, then midpoints of element sides can be connected to create four equilateral triangles from each original triangle (fig. 7*A*, dashed lines). The additional nodes, which appear on the midsides of elements that were not subdivided, are then linked by an element side to the

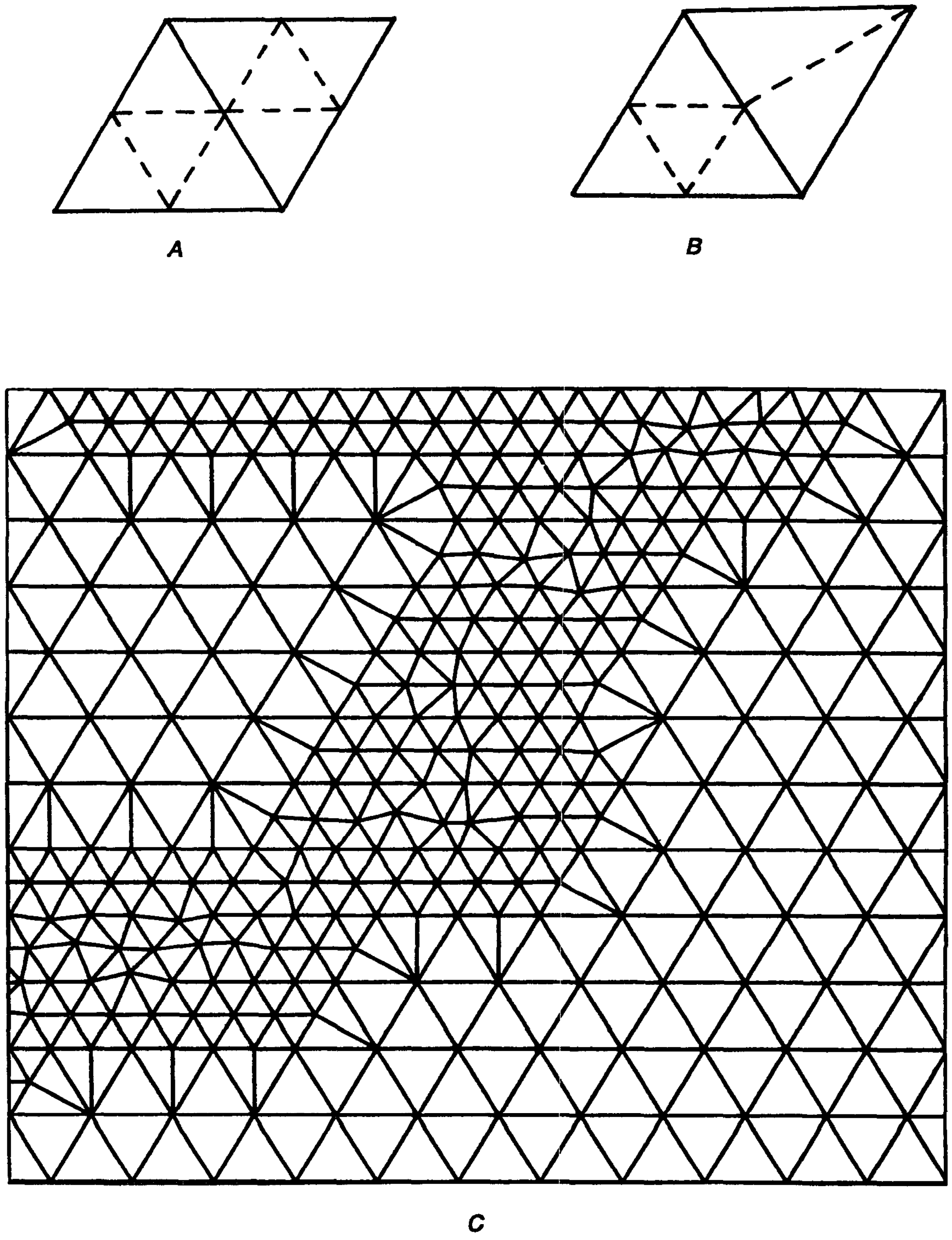


Figure 7.—Subdivision of elements by (A) connecting midpoints of element sides and (B) connecting midpoint to a vertex. (C) Finite-element mesh containing element subdivision.

opposite vertex of the larger element (fig. 7B). Subdivision of the formerly unchanged element in this manner creates two elements that are 30–60–90-degree triangles. These concepts of element subdivision are demonstrated in figure 7C where the coarse mesh is finely discretized near a river. Note that nodal locations along the river have been moved from the positions that resulted from the element subdivision. By carefully adjusting nodal locations, all angles in the mesh can range from 30 to 90 degrees.

For axisymmetric-cylindrical (radial) flow, the mesh can be designed according to the examples given in the section "Axisymmetric Flow", or according to the mesh used in the radial-flow problems given in Cooley (1992). Depending on aquifer geometry and subsurface lithology, a regular triangular mesh also can be used to discretize the aquifer region in cross section or in axisymmetric flow. However, care must be taken when designing a mesh in axisymmetric coordinates to ensure that the symmetry of the flow problem is represented by the mesh. (See section "Axisymmetric Flow.")

Identifying Discretization Error

The ability of the subdivision (discretization) scheme to accurately approximate the true hydrologic response to stress on the ground-water system is the ultimate test of adequacy in the design of the finite-element mesh. Aside from errors in the characterization of hydrologic properties and in the measurements used to check model validity, discretization error can undermine the computed solution and prevail throughout the study if it is undetected and not corrected. Discretization error may be manifested by the inability of the mesh to represent changing hydraulic gradients in the vicinity of a pumped well or discontinuity in hydrologic properties. Violation of certain mesh-construction rules, described in the previous section, could cause less-than-adequate approximations of hydraulic head within elements. In some cases, discretization error may be subtle, and the user may attempt unsuspectingly to compensate for these by adjusting hydrologic properties during the calibration process or by questioning the accuracy of measurements. In other cases, discretization errors may not be so subtle. In either case, the user must attempt to identify and minimize discretization error during the initial stages of mesh construction.

The process of identifying discretization error in a finite-element mesh is straightforward: the user tests whether refinement of the mesh creates acceptable or unacceptable changes in the computed solution. Mesh refinement involves changing element sizes or shapes, or both, and testing requires determining the effects of these changes on the simulation results. This test can be performed either on part of the aquifer region or on a small, prototype area that contains hydrologic properties and stresses that are characteristic of the aquifer region and the problem. Results of identical simulations that use different finite-element meshes can be compared to determine if the mesh refinements improved the computed solution. For example, if ground-water pumping was simulated under transient conditions, then the computed potentiometric surfaces resulting from fine and coarse discretizations are compared. The mesh that provided improved definition of the potentiometric surface, usually the finer mesh, is preferred over the other mesh. The mesh that provided a more acceptable solution is then further discretized and the simulation is repeated. Testing is concluded when additional mesh refinement does not improve the solution.

Discretization error that is associated with the shape of the finite elements can be identified by inspecting the mesh, if it is small, or by using a computer program to check angles between element sides. A test simulation can be performed after reconstructing part of the mesh that is suspect. Parts of the mesh that contain thin elements or elements that have obtuse angles can be reconstructed to eliminate these occurrences. The simulation is performed with the reconstructed mesh and the results are compared with the previous simulation. A brief discussion of the effects of obtuse angles and thin elements on the finite-element representation of hydraulic head is given in the previous section.

Node Numbering and Determining Bandwidth

After constructing the finite-element mesh, nodes are numbered so that data can be entered. Although node numbering can be arbitrary, certain numbering conventions will improve the computational efficiency of MODFE. For example, by minimizing differences between node numbers in any element, terms used to form coefficients for the finite-element matrix equation (254) in Cooley (1992) will be stored as close together as possible in the computer program, allowing computations to be performed efficiently. The maximum difference between node numbers in an element that does not contain a specified-head node is used to allocate computer storage for the direct-solution method. In a patch of elements, the maximum number of connections from the node in the center of the patch to higher node numbers will determine the computer-storage requirements for each node and the number of storage locations that are overwritten by the iterative, conjugate-gradient method of solution. Allocation of computer storage and execution time is discussed in Torak (1993). Guidelines are given below that will permit a node numbering scheme to be developed that utilizes the computer-storage and computational attributes of MODFE effectively.

Techniques to Enhance Computational Efficiency

Numbering nodes to enhance the computational efficiency of MODFE is related directly to minimizing the reduced-matrix bandwidth and indirectly to minimizing the condensed-matrix bandwidth. The effects

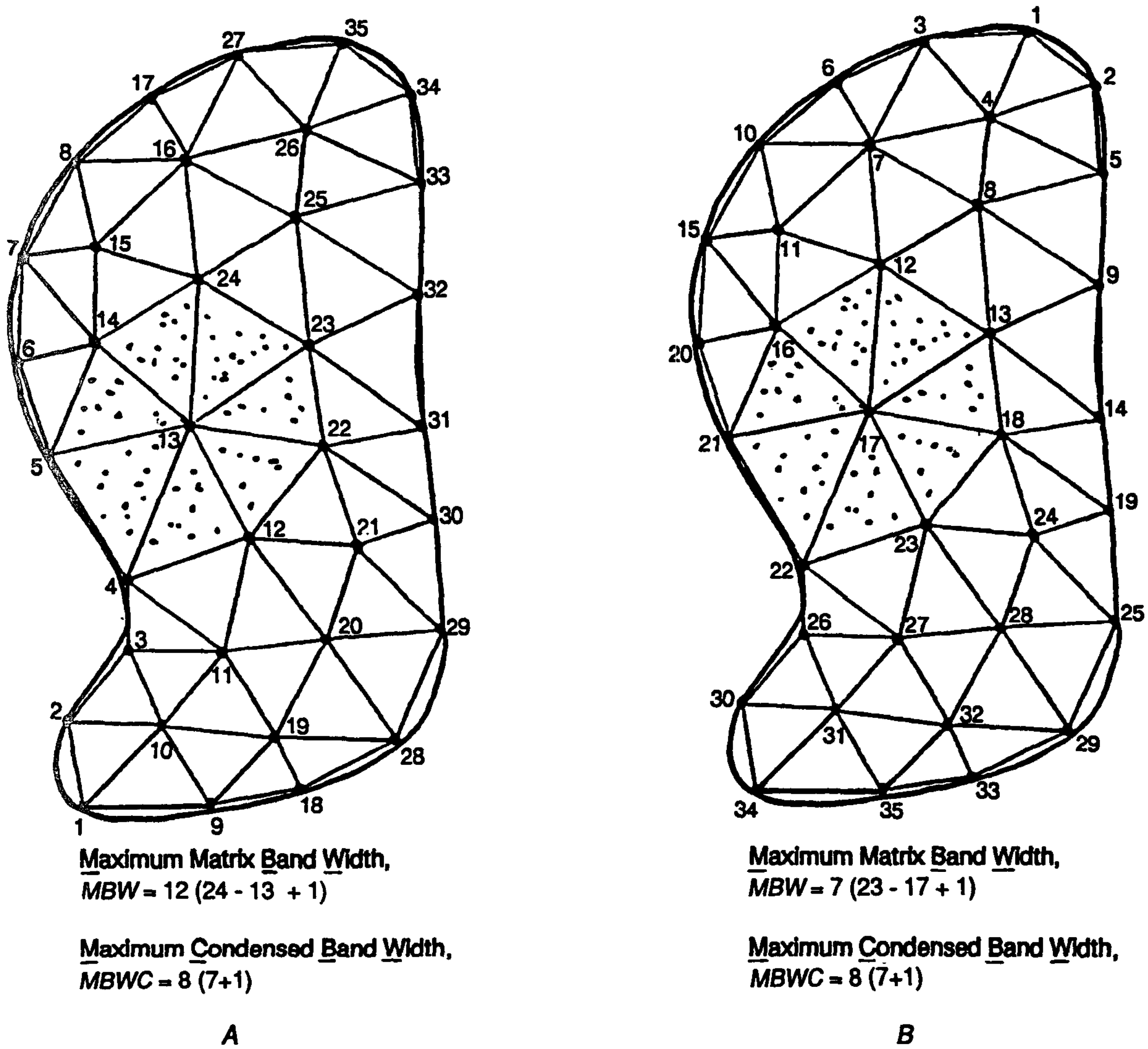


Figure 8.—Effect of node numbering on bandwidth for two identical finite-element meshes (*A*) and (*B*).

of node numbering on both bandwidth determinations are discussed here, and guidelines for bandwidth reduction are given. Definitions for the reduced matrix, condensed matrix, and bandwidths, are given in the following section for reference. In addition, the user can consult the appropriate sections of Cooley (1992) and Torak (1993) for a more detailed description of these terms.

The maximum difference between node numbers in any element and the reduced-matrix bandwidth usually can be minimized by numbering nodes along the shorter direction of the finite-element mesh. For example, given two identical meshes (fig. 8), the mesh that is numbered in the longer direction (fig. 8A) yields a maximum difference between node numbers of 11 (24–13) and a reduced-matrix bandwidth of 12. The mesh that is numbered in the shorter direction (fig. 8*B*) yields a maximum difference between node numbers of 6 (23–17) and a reduced-matrix bandwidth of 7. The reduced-matrix bandwidth is expressed in figure 8 and in MODFE as the program variable MBW. A specific definition of the reduced-matrix bandwidth is given in the following section.

The condensed-matrix bandwidth can be determined by inspecting the finite-element mesh and obtaining the maximum number of nodal connections to any node. Specifically, the condensed-matrix bandwidth is the maximum number of connections to higher-numbered nodes from the node in the center of a patch of elements, plus one. An approximation of the condensed-matrix bandwidth is obtained by counting element sides that are connected to a node in the center of a patch of elements and adding one to this sum. Usually, the value of the condensed-matrix bandwidth is influenced more by mesh design than by node numbering. However, the following example demonstrates that node numbering could be equally important as mesh design in minimizing the condensed-matrix bandwidth.

Consider the patch of elements corresponding either to node 13 in figure 8*A* or to node 17 in figure 8*B*. Although these nodes connect to seven other nodes in the patch of elements, the condensed-matrix bandwidth is determined to be five, as only four of the seven nodes have a higher node number than either 13 or 17, respectively. The condensed-matrix bandwidth is expressed in figure 8 and in MODFE as the program variable MBWC.

The condensed-matrix bandwidth usually is not affected by the size of the finite-element mesh. For regular triangular meshes, the condensed-matrix bandwidth may have a uniform value over the mesh, and the maximum value of MBWC may be determined easily by visual inspection. Irregular meshes make the determination of MBWC difficult; thus, care should be taken in the design of the mesh to avoid excessive element connections to a node.

Definitions

Reduced Matrix—The matrix of coefficients to finite-element matrix equation (254) in Cooley (1992), that results from eliminating equations corresponding to specified-head nodes. This elimination decreases the order of the matrix equation.

Reduced-Matrix Bandwidth—The maximum number of columns between and including the first and last nonzero entries in a row of the reduced matrix. The reduced-matrix bandwidth is estimated as the maximum difference between node numbers in any element that does not contain a specified-head node, plus one. The actual value of the reduced-matrix bandwidth is computed in MODFE by using index numbers to nodes (see sections "Reduced Matrix A" and "Reordering Finite-Element Equations for Solution" in Torak, 1993). It is used in MODFE to expand the condensed matrix prior to solution by the direct-solution method and to allocate computer storage (see section "Allocation of Computer Storage and Processing Time" in Torak, 1993).

Condensed (or Transformed) Matrix—A matrix consisting of the nonzero entries to the right of and including the main diagonals of the reduced matrix. The first column of the condensed matrix stores the main diagonal of the reduced matrix. Off diagonals of the reduced matrix are stored in subsequent columns to the right of the first column (main diagonal of the reduced matrix). This transformation uses the symmetry of the reduced matrix to decrease computer storage and increase the computational efficiency of MODFE.

Condensed-Matrix Bandwidth—The maximum number of nonzero terms across a row of the condensed matrix, determined from the finite-element mesh as the maximum number of higher-numbered nodes that connect to the node in the center of a patch of elements, plus one.

Solution Methods

Two methods for solving finite-element matrix equation (254) in Cooley (1992), are available in MODFE: a direct, symmetric-Doolittle method of triangular decomposition and an iterative, modified, incomplete-Cholesky, conjugate-gradient method (MICCG). Both methods are capable of providing numerical solutions that are limited only by the numerical precision of the computer, thus, each method should be considered equally plausible for the solution of a given matrix problem. The solution methods are interchangeable by replacing the set of subroutines and Fortran call statements pertaining to one method by the set pertaining to the other. Details of replacing solution methods are given in the section "Structure Diagrams for the Main Programs of MODFE", in Torak (1993).

Guidelines for selecting one solution method over the other are based mostly on computational efficiency, that is, the computer storage and time required for completing a simulation. The direct method can be used efficiently on aquifer problems containing a maximum of about 120 to 200 nodes. This limitation is only approximate, as the computational efficiency of the direct method (and the iterative method) is affected by the matrix bandwidths, described in the previous section. Thus, by minimizing the bandwidths, the efficiency of the solution methods is increased.

Both methods can be used to solve linear- or nonlinear-aquifer problems under steady-state or transient conditions. However, as explained in the section "Stopping Criteria" in Cooley (1992), the MICCG method may be more efficient than the direct method for solving nonlinear problems. Other guidelines and descriptions of data inputs for each solution method are given in the following sections.

Direct: Triangular Decomposition

The symmetric-Doolittle method of triangular decomposition solves matrix equation (254) in Cooley (1992) in a direct manner, without iteration. Details of the solution method are given in the section "Symmetric-Doolittle Method," in Cooley (1992). Briefly, the coefficient matrix A in equation (254) in

Cooley (1992) is factored into upper- and lower-triangular matrices that facilitate solution by a forward-elimination and back-substitution process. The method becomes less efficient computationally than the iterative MICCG method as the number of equations approaches the range of about 120 to 200.

Inputs for the direct-solution method consist of values for the reduced-matrix bandwidth and the condensed-matrix bandwidth. Descriptions of both bandwidths and methods for bandwidth minimization are given in the sections "Node Numbering and Determining Bandwidth" and "Techniques to Enhance Computational Efficiency." Values for the reduced-matrix bandwidth and the condensed-matrix bandwidth are input to MODFE as the program variables MBW and MBWC, respectively. Descriptions of data inputs to MODFE are given in the section "Input Instructions."

Iterative: Modified Incomplete-Cholesky Conjugate Gradient

The iterative method used to solve matrix equation (254) in Cooley (1992) is the modified incomplete-Cholesky conjugate-gradient (MICCG) method. Details of the MICCG method are given in the section "Modified Incomplete-Cholesky Conjugate-Gradient Method" in Cooley (1992), and references are given in that section for additional information about the method. In general, the coefficient matrix A of equation (254) in Cooley (1992) is split, approximately, into two matrices M and N. The factorization of M and the iterative scheme used in the MICCG method are essential to the computational efficiency of the method.

Guidelines for selecting the MICCG method over the direct method are based on the relative amounts of computer storage and execution time required for each method to solve a given aquifer problem. As stated in the previous section, the MICCG method is faster than the direct method for solving aquifer problems containing more than about 120 to 200 nodes. My experience with both methods on regular triangular meshes ranging in size from about 120 to 15,000 nodes (or equations) is that the MICCG method is about 35 to 70 percent faster than the direct method, with the larger meshes producing the greater savings in execution time.

The MICCG method uses less computer storage than the direct method when applied to meshes containing more than about 120 nodes. For meshes having fewer than about 120 nodes, the direct-solution method may use less computer storage (and time). However, the larger the aquifer problem, in number of nodes, the more favorable is the selection of the MICCG method for solution. For example, the MICCG method required about 40 percent less computer storage than the direct method for a mesh containing about 1,100 nodes. As described in previous sections, the computational efficiency of either method is related to mesh design and to the bandwidth determinations; however, the direct method is affected more by these factors than the iterative method.

The iterative MICCG method may have computational advantages over the direct method for solving nonlinear steady-state problems. As described in the section "Stopping Criteria" in Cooley (1992), the number of MICCG iterations (on an 'inner' iteration loop) can be decreased by the appropriate selection of a closure tolerance. Thus, the reduced computation time caused by taking only a few (inner) MICCG iterations may be less than the time needed to obtain a solution by the direct method. The numerical accuracy of the nonlinear solution is controlled by another closure tolerance that is placed on the outer (water-table) iterations, which are independent of, but contain, the solvers. Details about selecting a closure tolerance for the water-table iterations are given in the section "Nonlinear Conditions."

Inputs for the MICCG method consist of values for the maximum number of iterations and the closure tolerance. The maximum number of iterations is represented in MODFE as the program variable NIT, and the closure tolerance is represented as the variable TOL. Because the reduced-matrix bandwidth, MBW, is not used for the MICCG method, the maximum number of iterations, NIT, replaces MBW in the data inputs. (See section "Input Instructions.") Values for NIT are problem dependent, and usually range from about 10 to 20, for linear problems, to about 100 for nonlinear problems. The number of iterations needed for solution also is dependent on the value of the closure tolerance, TOL.

Values for the closure tolerance, TOL, are selected small enough to ensure an acceptable solution of hydraulic heads and flow rates, yet large enough to avoid excessive iteration. Usually, values for TOL range from about 0.001 to 0.0000001. The larger value is about an order of magnitude smaller than the error associated with water-level measurements, and the smaller value is near the limit of numerical accuracy for single-precision computations by most computers. For nonlinear steady-state problems, TOL should be set large enough so that closure is obtained within a few MICCG iterations. As described above, the accuracy of a nonlinear solution is controlled by an additional closure criterion for water-table iterations. Details about the operation of water-table iterations and selection of this closure criterion are given in the section "Nonlinear Conditions" and in the section "Nonlinear Case" in Cooley (1992).

Although MICCG is an iterative method, values for iteration parameters are not input to MODFE. Instead, values for iteration parameters are computed automatically within the solver subroutine at the time of execution. Thus, with the appropriate value for the closure tolerance, TOL, the MICCG method will produce a solution efficiently that is as numerically accurate as the direct method.

Aquifer-Simulation Capabilities

MODFE contains the following aquifer-simulation capabilities:

- transient or steady-state conditions,
- nonhomogeneous and anisotropic flow where directions of anisotropy change within the model region,
- vertical leakage from a semiconfining layer that contains laterally nonhomogeneous properties and elastic storage effects,
- point and areally distributed sources and sinks,
- specified-head (Dirichlet), specified-flow (Neumann), and head-dependent (Cauchy-type) boundary conditions,
- vertical cross sections,
- axisymmetric-cylindrical flow,
- confined and unconfined (water-table) conditions,
- partial drying and resaturation of a water-table aquifer,
- conversion between confined- and unconfined-aquifer conditions,
- nonlinear-leakage functions (for simulating line, point, or areally distributed sources and sinks),
- changing stresses and boundary conditions on a stress-period basis, time-step basis, or both, and
- zoned input of hydraulic properties and boundary conditions.

The simulation capabilities listed above are described in sections of this report with regard to the physical processes that describe the hydrologic phenomena and to their implementation in MODFE. Mathematical symbols used by Cooley (1992) to describe these capabilities are replaced here by program variables which are contained in MODFE. Brief descriptions of data inputs for these simulation capabilities are given to enable the user to link the hydrologic phenomena to the mathematical representation in MODFE. Detailed data-input instructions are given in the section "Input Instructions."

Nonhomogeneity and Anisotropy

Two-dimensional ground-water flow in aquifers that exhibit nonhomogeneity and (or) anisotropy with regard to hydrologic characteristics (either hydraulic properties or boundary conditions) can be simulated by MODFE. Nonhomogeneous conditions are represented in MODFE by inputting distinct values for hydrologic characteristics by element or by node. The following hydrologic characteristics are input to MODFE by element:

- aquifer hydraulic conductivity or transmissivity,
- rotation angle for anisotropy in aquifer hydraulic conductivity or transmissivity,
- vertical hydraulic conductance of confining bed,
- aquifer storage coefficient and (or) specific yield, and
- unit rate of areally distributed stress.

The following hydrologic characteristics are input to MODFE by node:

- volumetric flow rates at point sources and sinks,
- aquifer thickness and altitude of aquifer top (for water-table simulations), and
- specified-flux, or head-dependent (Cauchy-type) flux boundary conditions (linear and nonlinear conditions).

Although MODFE can represent nonhomogeneity in hydrologic characteristics with element or nodal inputs, rarely are these characteristics known with enough detail throughout the aquifer region to permit the input of distinct values either by element or by node. Usually, values for hydrologic characteristics are generalized into zones, which contain either groups of elements or groups of element sides, from which element and nodal inputs can be obtained. A discussion of preparing data for input to MODFE by zone is given in the section "Hydraulic-Property and Boundary-Condition Zones."

Anisotropic flow, where principal values of hydraulic conductivity (or transmissivity) and principal directions vary within the aquifer region (fig. 9), can be represented by MODFE for simulation. Variation of principal values within the aquifer region is represented in the same manner as described above for nonhomogeneity; separate inputs are made for the principal values in the x and y directions by element or by zone. The program variables used to represent the principal values of transmissivity (or hydraulic conductivity) in the data input are XTR and YTR for the x and y directions, respectively. Details of these inputs by element or by zone are given in the section "Hydraulic-Property and Boundary-Condition Zones."

Anisotropic conditions of varying principal directions within the aquifer region are represented easily in MODFE by transforming the coordinates within elements that require change. The global x-y coordinate system that is used to represent nodal locations is rotated orthogonally within an element or a zone to a

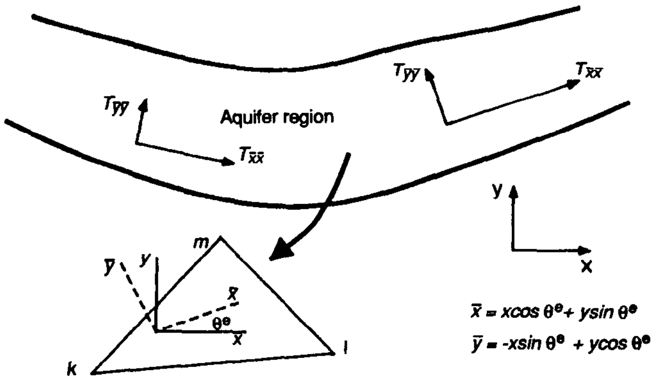


Figure 9.—Anisotropic flow conditions and nomenclature used to transform coordinates to local, $\bar{x}$-$\bar{y}$ system.

local $\bar{x}$-$\bar{y}$ system that coincides with the principal directions along which principal values of transmissivity (or hydraulic conductivity) have been determined. This transformation causes the symmetric transmissivity (or hydraulic conductivity) tensor of equation (1) to contain zero values for the off-diagonal or cross-product terms T_{xy} and T_{yx} (or K_{xy} and K_{yx}), and leaves the principal values $T_{\bar{x}\bar{x}}$ and $T_{\bar{y}\bar{y}}$ (or $K_{\bar{x}\bar{x}}$ and $K_{\bar{y}\bar{y}}$) in the tensor to characterize transmissivity (or hydraulic conductivity) in that part of the aquifer region. Usually, the principal values are determined by analyzing aquifer-test data. As described above, the principal values are input to MODFE by zone, and are represented by the program variables XTR and YTR for $T_{\bar{x}\bar{x}}$ ($K_{\bar{x}\bar{x}}$) and $T_{\bar{y}\bar{y}}$ ($K_{\bar{y}\bar{y}}$), respectively.

The rotation angle, θ^e, is the angular displacement of the local, $\bar{x}$-$\bar{y}$ system from the global x-y system, in degrees, measured counterclockwise. The rotation is performed automatically in MODFE by zone, according to the expressions shown in figure 9. If θ^e is zero, then no rotation is necessary. Because a zone could consist of only one element, it is possible to rotate elements individually. The rotation angle is represented in MODFE as the program variable ANG, and is input by zone along with the other zoned inputs listed above.

Steady Vertical Leakage

Steady vertical leakage through a confining bed from a source layer situated above or below an aquifer can be simulated by MODFE (fig. 10*A*). In addition to this application, steady vertical leakage can be used to represent leakage through the bottom of a wide, partially penetrating riverbed or lakebed, provided that the aquifer head is situated above the altitude of the bottom of the riverbed or lakebed (fig. 10*B*,*C*). (The condition in which the aquifer head sometimes lies below the altitudes of the bottom of an overlying confining bed and below the bottom of the riverbed or lakebed is represented in MODFE by a nonlinear leakage function, which is discussed in the section "Nonlinear-Leakage Functions"). The leakage is called steady because storage effects in the confining bed are not considered in the flow-rate computations; the flow rate from steady vertical leakage is dependent only on the vertical hydraulic conductance of the confining bed and on the head difference between the aquifer and the source layer. (The vertical leakage or yield of water stored elastically in a confining bed (transient leakage) is discussed in the following section.) Steady vertical leakage is represented in MODFE by the term R(H − h) in equation (1), where R is vertical hydraulic conductance of the confining bed, H is source-layer head, and h is aquifer head (fig. 10).

Values of vertical hydraulic conductance are represented in MODFE by hydraulic-property zone, and are identified as the program variable VLC. Values for VLC can be computed from the zone representations of confining-bed thickness and vertical hydraulic conductivity. Usually, vertical hydraulic conductivity is not known with as much detail as is thickness; thus the aquifer region may contain only a few conductivity zones in comparison with the number of thickness zones. Preparation of zone inputs to MODFE are discussed in the section "Hydraulic-Property and Boundary-Condition Zones." Depending on the lithology and application of steady vertical leakage, the confining bed may consist of an entire geologic unit, several units, or part of one unit.

The source-layer head is input to MODFE by node, and is represented as the program variable HR. As shown in figure 10, the source-layer head can represent the head in an aquifer situated above or below the aquifer to be simulated, the head in a perched layer within the confining bed, or a lake or stream level.

The source-layer head, HR, may be changed with time during the simulation. Details of changing HR with time are given in the section "Changing Stresses and Boundary Conditions With Time."

Vertical Leakage of Water Stored Elastically in a Confining Bed

MODFE has the ability to simulate nonsteady vertical leakage of water stored elastically in a confining bed (transient leakage). The confining bed either can be an areally extensive geologic unit that completely or partially confines the simulated aquifer, or it can be a smaller unit, such as a riverbed or lakebed (fig. 10). Transient-leakage effects occur when nonsteady vertical hydraulic gradients are created and then dissi-

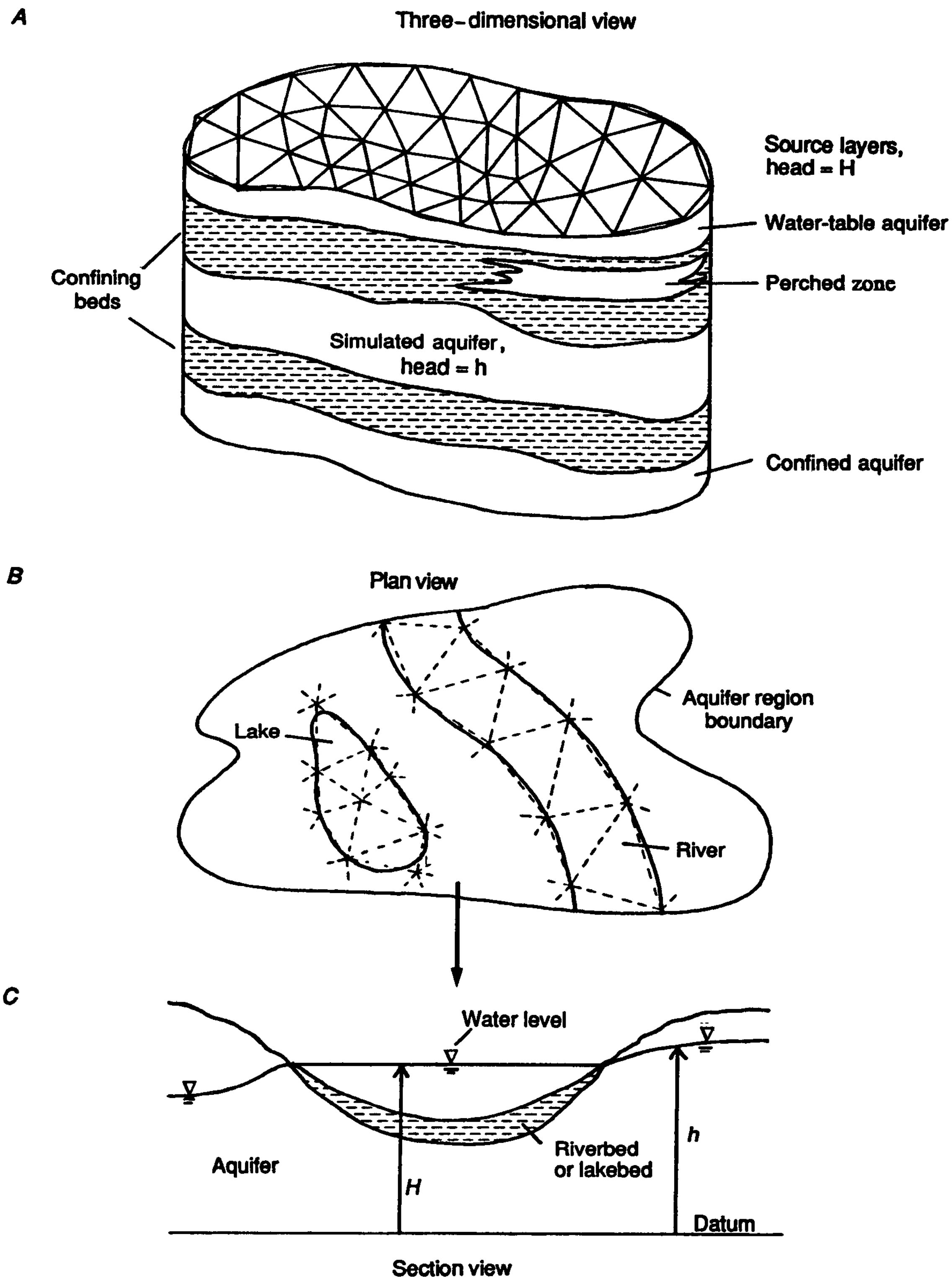


Figure 10.—Examples of steady vertical leakage (*A*) from source layers and (*B*) and (*C*) from areally extensive surface-water features.

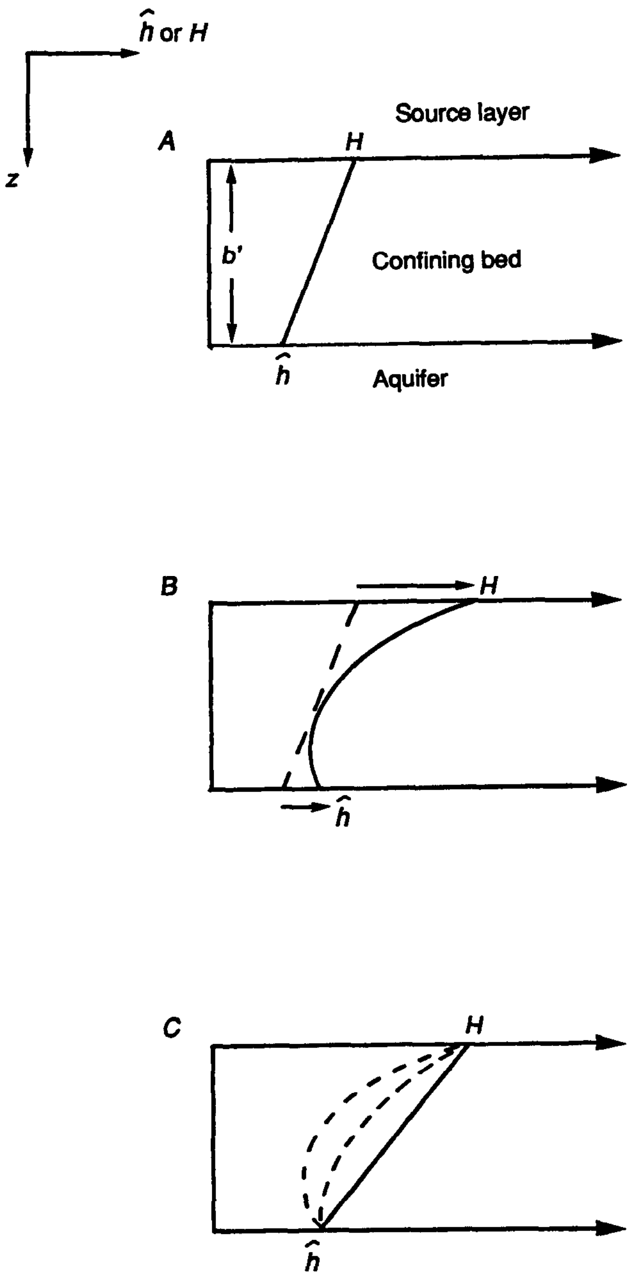


Figure 11.—Vertical head distributions in a confining bed for (*A*) initial, steady-leakage conditions at time $t=t_n$; (*B*) transient conditions following changes to aquifer head, $\hat{h}$, and source-layer head, H, at $t=t_n+\Delta t_{n+1}$; and (*C*) re-establishment of steady-leakage conditions at $t>>t_n+\Delta t_{n+1}$.

pate within the confining bed in response to head changes at the confining-bed boundaries (fig. 11*A*,*B*). These head changes commonly are the result of changing stresses in the aquifer, such as changing or initiating well pumpage, or changing fluxes or controlling heads to aquifer-boundary conditions. If the conditions causing the head changes at the confining-bed boundaries are fixed in time, then the nonsteady hydraulic gradients will adjust gradually until a new steady-state condition is established (fig. 11*C*). Depending on the direction of head changes at the boundaries of the confining bed, water is either released from or taken up by storage in the confining bed. This process creates nonsteady (transient) leakage rates across the boundaries of the confining bed that differ substantially from those that are calculated if yield from storage effects in the confining bed is neglected (steady leakage).

The existence and magnitude of transient-leakage effects on the flow system is related to the physical and hydraulic properties of the confining bed, the magnitude of head changes at its vertical boundaries, and the relative time over which the aquifer head is simulated. Transient-leakage effects are not present during steady-state simulations because steady-state ground-water flow is independent of time, whereas transient effects are time dependent. For nonsteady-state conditions in the aquifer, flow rates from transient leakage can vary within orders of magnitude either at a given location during the simulation period or throughout the model area at a given instant in time.

The importance of transient-leakage effects on the vertical leakage rate can be determined by evaluating a term called dimensionless time. Dimensionless time is expressed in the equation development of Cooley (1992) as the product of simulation time and the confining-bed properties of thickness, vertical hydraulic conductivity, and specific storage. The confining-bed properties are given as γ_i in equation (167) in Cooley (1992), and time is either the total simulation time or the time-step size. Dimensionless time is used in the expressions for approximating transient-leakage flux during a time step. In general, these expressions contain two infinite series that are approximated with exponential functions over dimensionless time to obtain the transient-leakage flux (see development leading to equation (198) in Cooley, 1992). The series and the approximating functions indicate that transient-leakage effects are an important part of the leakage rate when dimensionless time is less than about 0.1. For values of dimensionless time greater than about 0.1, the transient-leakage effects can be neglected, as contributions to the leakage rate from the series and the exponential functions are small, and the leakage rates can be approximated by steady vertical leakage (discussed in the previous section).

The formulation for γ_i in equation (167), (Cooley, 1992), and the simulation time can be used to evaluate the effects of transient leakage on the vertical leakage rate. The term γ_i indicates that, for a given simulation time, transient-leakage effects will be present and persist during the simulation (values of dimensionless time less than 0.1) if the confining bed is thick and has a low vertical hydraulic conductivity than if it is thin and has a higher vertical hydraulic conductivity. Similarly, for a given set of confining-bed properties, transient-leakage will have more of an affect on the computed hydraulic head for simulations of short duration than for simulations of long duration. The user can determine the importance of transient leakage on the vertical leakage rate by computing γ_i for each zone that contains a different set of confining-bed properties, and multiplying the γ_i values by simulation time to compute dimensionless time.

Transient-leakage effects usually are larger than steady-leakage effects during the time immediately following changes to stresses or boundary conditions, as nonsteady vertical gradients are at their maximum at this time. The transient effects gradually diminish with time as stresses and boundary conditions remain constant during the simulation, leaving only steady leakage to characterize the flow rate across the confining bed. However, it is possible for nonsteady vertical hydraulic gradients to persist throughout the simulation period, making transient-leakage effects an important water-budget component for the aquifer.

The transient-leakage approximation in MODFE is a linear process that is applied to nonsteady confined- and unconfined-flow problems. It may be required, however, when simulating other (nonlinear) features in combination with transient leakage, as shown in the structures of the main program in the section "Program Structures and Lists of Main Programs," (Torak, 1993).

Inputs for transient leakage consist of the vertical hydraulic conductivity and specific storage of the confining bed. Also, the inputs of vertical hydraulic conductance (VLC) and the source-layer head (HR), which were described in the section "Steady Vertical Leakage", are required for simulating transient leakage. The values for VLC and HR are used in computations for steady and transient leakage, and are input once for both formulations. The vertical hydraulic conductivity and specific storage are represented in MODFE as the program variables VCON and SPST, respectively, and are input by hydraulic-property zone. Details of preparing data for input by hydraulic-property zone are given in the section "Hydraulic-Property and Boundary-Condition Zones."

Areally Distributed Sources and Sinks

Areally distributed stresses, such as precipitation, infiltration from applied recharge, or a constant evapotranspiration rate, can be simulated by MODFE. These stresses are represented in equation (1) and in figure 12 as the term W, which is the unit areal recharge or discharge rate (volumetric flow rate per unit area). W has dimensions of [length/time].

Values for the unit rate are represented in MODFE as the program variable QD, and are input by hydraulic-property zone. Within MODFE, the unit rate is multiplied by one third of the element area, $(1/3)\Delta^e$ (fig. 12), so the volumetric flow rate for the areally distributed stress can be applied to each node in the element. Details of these computations are given in the section "Areally Distributed Sources and Sinks" in Torak (1993).

The unit rate of areally distributed stress is allowed to vary with time in MODFE. New unit rates can be input for a time step or a stress period. Details of changing areally distributed stresses with time are given in the sections "Input Instructions" and "Changing Stresses and Boundary Conditions with Time" (Torak, 1993).

Point Sources and Sinks

Point sources and sinks, such as wells and constant-flow drains, can be simulated by using MODFE. These features are represented in equation (1) as the term P. A specified-flow condition also can be simulated as a point source or sink; however, it usually is represented by the specified-flow part of a Cauchy-type boundary, which is described in the section "Boundary Conditions."

Point sources and sinks are represented in MODFE by assuming the stress is located at a node. Thus, nodes are positioned at locations of the stress or anticipated stress when the mesh is designed. The flexibility in mesh design allows nodes to be placed at the location of the point stress for most problems.

If nodes cannot be positioned at the location of the point stress within the model region, then the point stress can be distributed to the nodes of the element in which the stress is contained by assuming linear variation of the stress within an element. The point stress is apportioned to the nodes of an element in the same manner as hydraulic head at any location within an element is defined by the head at the nodes (see equation (8) in Cooley, 1992). Values for the coordinate functions, N_i, given by equations (9)–(11) in Cooley (1992), are used to weight the point stress proportionately at the three nodes. These values can

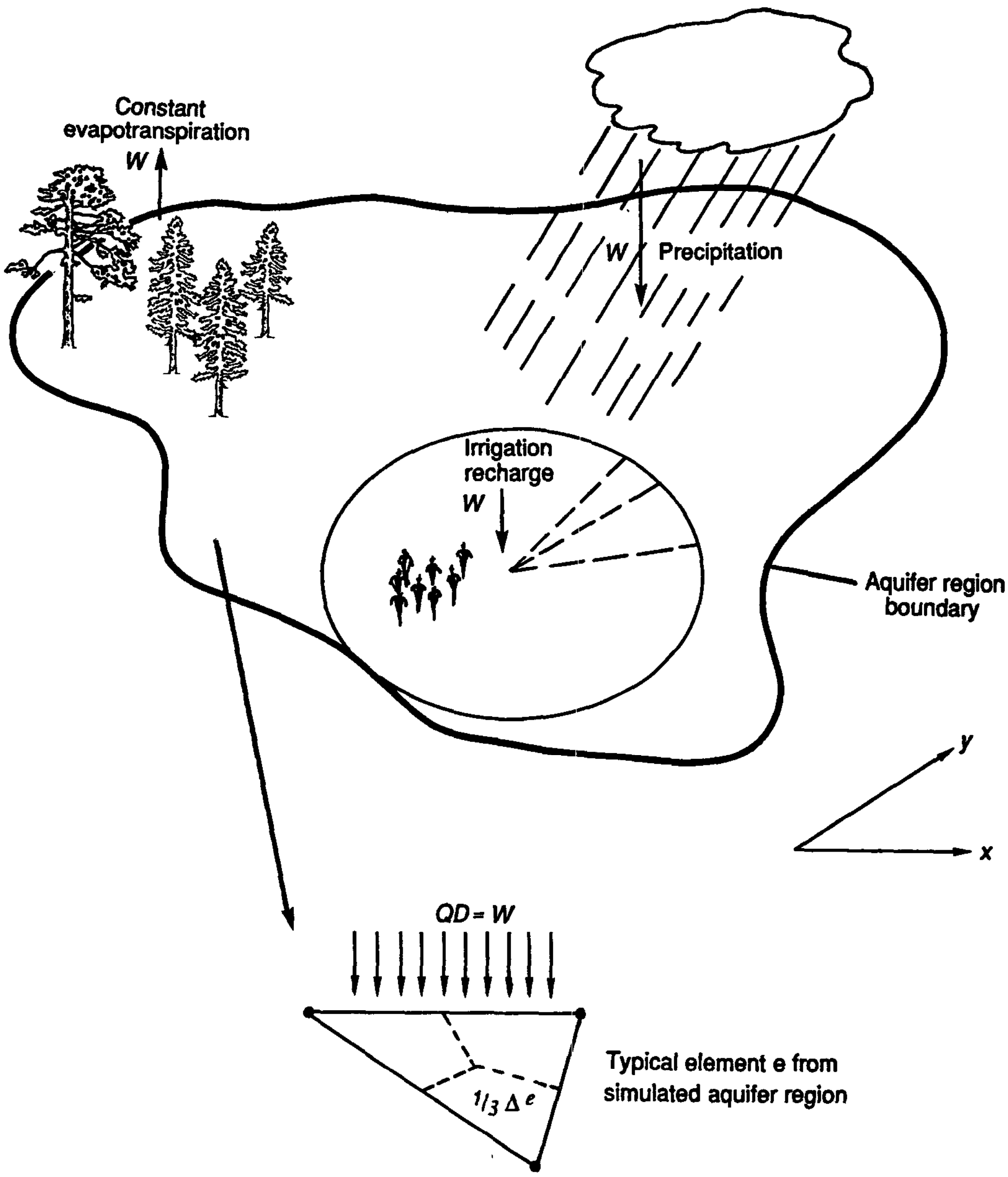


Figure 12.—Areally distributed stress on aquifer region and representation by a typical element.

be obtained by hand calculations or from a small computer program by utilizing the x-y coordinates of the point stress and of the nodes defining the element. However, care should be taken in applying this technique to relatively small elements so that the geometry of the aquifer problem is preserved.

Data inputs for point sources and sinks consist of the node number where the stress is located and the volumetric flow rate. The volumetric flow rate is represented in MODFE as the program variable QWEL, and the node number is represented as the program variable J. Prior to these inputs, the number of point sources and sinks that are simulated initially is input as the program variable NWELS.

The values and locations of point stresses can be changed before any time step or stress period of the simulation. Details of the data inputs required for changing point stresses with time are given in the section "Changing Stresses and Boundary Conditions with Time." Instructions for establishing stress periods and time-step sizes are given in the section "Selecting Stress-Period and Time-Step Sizes."

Initial Condition of Hydraulic Head

Equation (1) is subject to the initial condition of hydraulic head given by equation (5) in Cooley (1992), or

$$h(x,y,t) = h_o(x,y), \tag{2}$$

where $h_o(x,y)$ is the head or reference altitude at a node located at a point (x,y) in the aquifer at the start of the simulation period (at t=0). Values for $h_o(x,y)$ can be the altitude of the potentiometric surface, the water-table, or some other value to which the computed heads are referenced. For axisymmetric coordinates, $h_o(x,y)$ is replaced by $h_o(r,z)$ in equation (2). Head is represented in MODFE by the program variable H. The initial values of hydraulic head are input at each node. In subsequent computations, H represents different values before it becomes the computed solution. For simulation of transient conditions, H represents first the average hydraulic head during the time step, and then the head at the end of the time step. This latter value is used as the initial condition of head for the next time step or as the computed solution. For steady-state simulations, H represents either the head at the end of the current iteration or the initial condition at the beginning of the next iteration.

Boundary Conditions

MODFE can simulate three types of boundary conditions to equation (1): specified head (Dirichlet), specified flux (Neumann) and mixed (Cauchy). Each type is represented by equation (4) of Cooley (1992). The specified-flux and specified-head conditions are special cases of the mixed condition (see Bear, 1979, p. 117–120 for a description of these boundary conditions). Computations for the specified-flux and mixed conditions are combined in MODFE; the specified-head condition is formulated separately from the other two conditions. Because of the relation between specified-flux and mixed conditions, they are referred to as Cauchy-type boundaries in this documentation. Examples showing the application of specified-head and Cauchy-type boundaries to aquifer problems and their implementation in MODFE are given in the following sections. The Cauchy-type condition is separated into two components; specified flux and head-dependent flux. Each component is discussed separately as they have different hydrologic applications to ground-water flow.

Specified Head

The specified-head, or Dirichlet, condition is one in which hydraulic head is known at an aquifer boundary and is fixed during the simulation. Specified-head boundaries occur when the aquifer region is in direct hydraulic connection with a body of water whose level is either constant or varies in a predictable manner over time and is not affected by stresses in the aquifer. A specified-head condition also can exist along the boundary of the model region (but not necessarily the aquifer boundary) where the hydraulic head is known and is unaffected by stresses within the aquifer.

Examples of specified-head boundaries are shown in figure 13. These are: (1) a river or (2) a lake, both fully penetrating the aquifer and having a known water level, (3) an ocean-discharge boundary where the freshwater head at the shore is known, and (4) the boundary of the simulated region, where the head along the boundary is known and is not expected to vary in response to stresses in the aquifer. Although specified-head boundaries are suitable for representing the hydrologic features shown in figure 13, other boundary conditions, discussed in later sections, may be more appropriate for simulating these features than the specified-head condition.

Specified-head boundaries are represented in MODFE by assigning boundary heads to nodes. Values for specified-head nodes are input as the program variable HB (fig. 13). Prior to the input of HB, the number of nodes used to simulate specified-head conditions is input as the program variable NHDS. For steady-state simulations, at least one specified-head condition (node) is required in order to obtain a unique solution to the aquifer problem. (This mathematical requirement also can be satisfied by at least one head-dependent, Cauchy-type boundary, described in a following section.)

Head values on specified-head boundaries can be constant for the entire simulation period, or they can vary as functions of time. Descriptions of the data inputs and the program structure required to change boundary conditions with time are given in the section "Changing Stresses and Boundary Conditions with Time," and in the section "Changing Stresses and Boundary Conditions with Time" in Torak (1993).

Specified Flux

A specified-flux, or Neumann, condition is one in which the normal component of ground-water flow is known across a boundary. This is a special case of the mixed (Cauchy) condition, and is obtained from equation (4) in Cooley (1992) by setting $\alpha = 0$. The specified-flux condition is defined in equation (4) as the term q_B [length^2/time], which represents a unit discharge rate across the boundary, or volumetric flow rate per unit length of boundary (fig. 14). It is used where the volumetric flow rate, $Q_B = q_B L$, is pre-

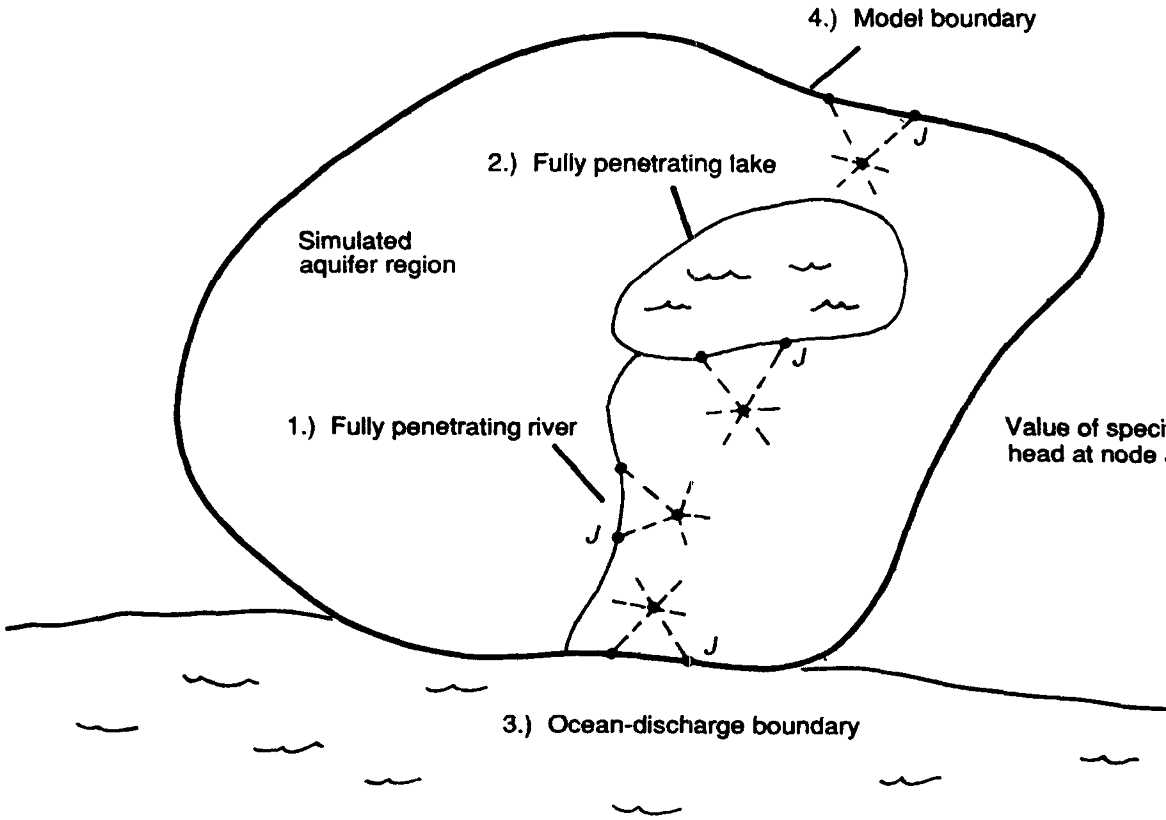


Figure 13.—Applications of specified-head boundaries for a simulated aquifer region and nomenclature used in MODular Finite-Element model (MODFE).

scribed across internal or external boundaries of the model region during simulation, where L is the length of the boundary.

Examples of specified-flux boundaries are regional flow across an external model boundary, constant-flow drains, and flow to a well in cross section (fig. 14). Specified fluxes from regional flow usually result from selecting a model area that is smaller than the areal extent of the aquifer. For this case, the specified-flux condition accounts for flow rates across model boundaries that emanate either from outside the model area or from within the model area and are not expected to change during the simulation. Other examples of specified-flux boundaries in aquifer cross sections or in axisymmetric (radial) problems are given in the sections "Cross Sections" and "Axisymmetric Flow." A special case of a specified-flux condition is a zero-flux, or impermeable, boundary.

The manner in which the unit discharge rate q_B is obtained for input to MODFE varies with each application of the specified-flux condition. For most applications (fig. 14*A*,*B*), q_B is obtained by dividing the volumetric flow rate Q_B by L, the length of the boundary along which Q_B is known. However, as discussed below, q_B requires other computations in order to represent the effects of areally distributed recharge over the region outside the simulated model area. Zero-flux or impermeable boundaries do not require specification of q_B as they are represented automatically along external boundaries of the model area.

The computation of the unit discharge rate q_B to represent areally distributed recharge located outside the model area depends on the geometry of the model boundary (fig. 14*C*). For curved model boundaries, q_B can be approximated by flow within a converging stream tube, whereas for relatively straight boundaries, q_B can be approximated by flow within a parallel stream tube. For a parallel stream tube, the unit discharge is computed as $WL_s/2$, where W is the unit rate of areally distributed recharge (or discharge), from equation (1), and L_s is the distance from the model boundary over which W is applied. For a converging stream tube, q_B is given by

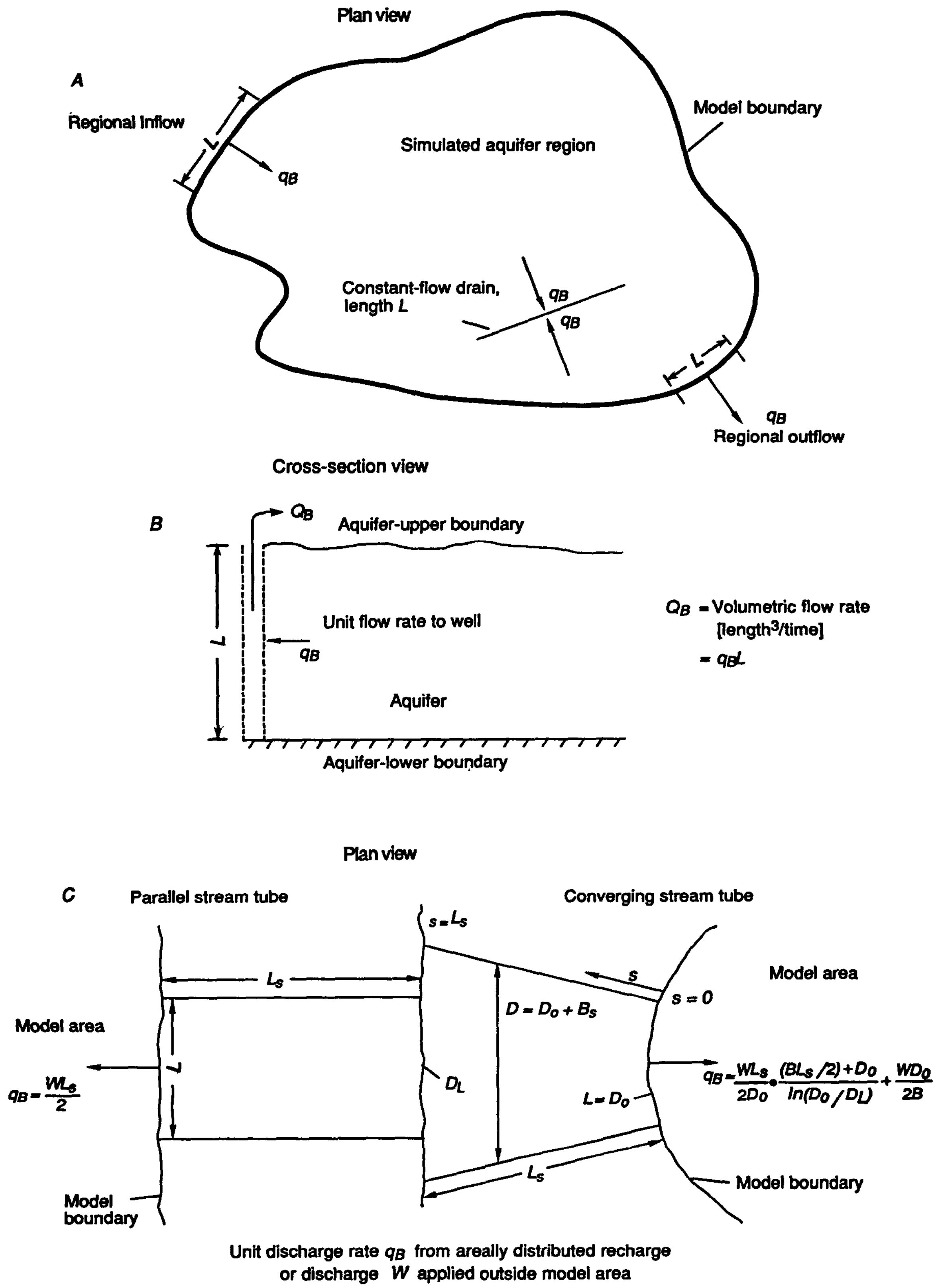


Figure 14.—Specified-flux boundaries representing (*A*) regional inflow and outflow and internal drainage; (*B*) constant flow to a well in cross section; and (*C*) areally distributed recharge or discharge outside simulated aquifer region.

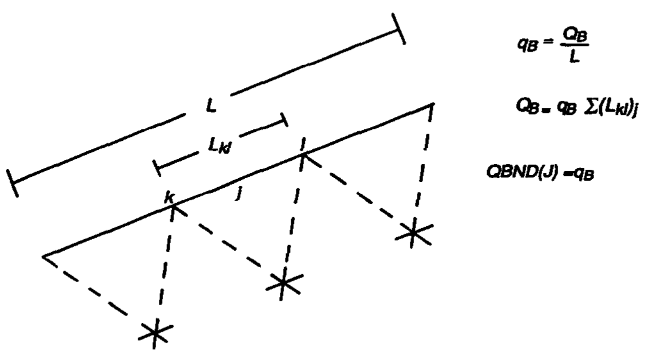


Figure 15.—Specified-flux boundary having length L and volumetric flow rate Q_B, subdivided by element sides j, and nomenclature used in MODular Finite-Element model (MODFE).

$$q_b = \frac{WL_s}{2D_o} \times \frac{\left(\frac{BL_s}{2}\right) + D_o}{\ln\left(\frac{D_o}{D_L}\right)} + \frac{WD_o}{2B} \tag{3}$$

where D_o and D_L are the widths of the stream tube at the model boundary and at the distance L_s from the boundary, respectively. The value of q_B accounts for an increased width, D, of the stream tube with distance, s, from the boundary. The rate at which D increases with s is given by B and is incorporated into the computation of q_B.

Specified-flux boundaries are represented in MODFE by element sides (fig. 15). Values for the unit discharge rate q_B are input to MODFE by boundary side j as the program variable QBND(J). Node numbers k and l defining element side j are input with QBND(J), respectively, as the program variables KQB(J) and LQB(J). Boundaries that use more than one element side to represent the total length L require separate input of QBND(J) for each side. Element sides containing the same values of QBND(J) can be grouped into boundary-condition zones to facilitate data input (see section "Hydraulic-Property and Boundary-Condition Zones").

The number of specified-flux boundaries (element sides) is summed with the number of head-dependent boundaries and is input to MODFE as the program variable NQBND. The value of NQBND is used to dimension storage locations and to control computations in MODFE that involve Cauchy-type boundaries. Programming details for these boundary conditions are given in the section "Specified-Flux Boundaries" in Torak (1993).

The unit discharge rate on specified-flux boundaries can vary as a function of time or be constant during the simulation period. Details on changing boundary conditions with time are given in the section "Hydraulic-Property and Boundary-Condition Zones."

Head-Dependent (Cauchy-Type) Flux

The head-dependent (Cauchy-type) condition relates the normal component of ground-water flow across a boundary to a head difference. The boundary condition is given by equation (4) of Cooley (1992) with $q_B = 0$. The head-dependent (Cauchy-type) flux yields steady-flow rates (no storage effects) across the boundary, in that the head difference, $H_B - h$ is related to the flow rate by a scalar function, α. The meaning of α varies depending on the application of the flux condition. However, α should not be set to a large value to create the specified-head (Dirichlet) condition as this condition is formulated differently in MODFE (see previous section). Because of this limitation on α, the true, mixed (Cauchy) condition is not represented in MODFE; instead, a Cauchy-type condition is formulated.

The head-dependent (Cauchy-type) flux has many applications to simulating ground-water flow (figs. 16–18). This boundary condition usually is needed when the model area is smaller than the physical extent of the aquifer, and regional flow occurs across model boundaries. Similarly, the head-dependent (Cauchy-type) flux allows ground water to flow across model boundaries in response to stresses, such as wells, that are operating near the boundaries within the model area. Aquifer drawdown is permitted at model boundaries by using this boundary condition. However, because the flow rate across the boundary is steady, the effects of storage or stresses outside the model area are not represented by the head-dependent (Cauchy-type) flux. Thus, model boundaries should be established so that only small percentages of the total drawdown from a well reach the boundary. (The specified-flux condition can be used to represent areally distributed stresses in the region outside the model area, as discussed in the previous section.)

Within the model region, head-dependent (Cauchy-type) fluxes can represent leakage to or from rivers, drainage ditches, fracture or fault zones, or other line-oriented features. In cross sections or in axisymmetric flow, head-dependent-flux conditions can represent lateral or vertical flows across model boundaries, such as regional flow, vertical leakage, or flow along fracture or fault zones. Applications of head-dependent (Cauchy-type) flux to cross-section and axisymmetric problems are described, respectively, in the sections "Cross Sections" and "Axisymmetric Flow."

Head-dependent (Cauchy-type) boundaries are represented in MODFE by element sides (figs. 16*B*, 17*B*,*C*, and 18). Values for α are required for each element side j on a boundary, and external heads H_B are defined for boundary nodes k and l. The term α is represented in MODFE by program variable ALPH(J). Nodes k and l defining element side j are represented, respectively, as the program variables KQB(J) and LQB(J). External or boundary heads H_B are represented as the program variables HK(J) and HL(J) for nodes k and l, respectively. Boundary sides that contain the same α value can be grouped into zones to facilitate data input. Details of establishing boundary-condition zones are given in the section "Hydraulic-Property and Boundary-Condition Zones."

The computation of α for simulating regional flow across model boundaries as head-dependent (Cauchy-type) fluxes is dependent on boundary geometry. Flow across highly curved model boundaries is approximated by a converging stream tube, and flow across relatively straight boundaries is approximated by a parallel stream tube (fig. 16*A*). For a parallel stream tube, the α value is computed as T/L_c, where T is the average transmissivity of the aquifer (or other porous material) between the model boundary and H_B, and L_c is the distance from the boundary to H_B. For a converging stream tube, α is computed as $TB/[D_o \times \ln(D_o/D_L)]$, where T is transmissivity as defined above, and D_o and D_L are the widths of the stream tube at the model boundary and at the distance L_c from the boundary, respectively. The value of α for a converging stream tube accounts for the increase in width, D, of the stream tube with distance, s, from the boundary. The rate of increase of D with s is given by B and is incorporated into the computation of α for the converging stream tube.

For simulating regional flow as a head-dependent (Cauchy-type) boundary, the external head H_B is located sufficiently far from the model boundary so that head changes at the boundary are not transmitted to H_B (fig. 16). Head changes over L_c are assumed to be linear; that is, steady hydraulic gradients are assumed to exist in the region between the model boundary and H_B.

To simulate leakage to or from a river, drainage ditch, or other line-oriented, leaky, surface-water feature (fig. 17), α is computed as the product of the vertical hydraulic conductance (vertical hydraulic conductivity divided by thickness) of the surface-water sediments and the width of the feature. The boundary head H_B is the surface-water level, such as river stage. These properties of the boundary condition are shown in figure 17*A*, where W_r is width and b_r is thickness. The term α is given by $(K_r W_r)/b_r$, where K_r is vertical hydraulic conductivity. In computations within MODFE, α is multiplied by the length of element side j on the boundary, L_{kl}, to give the area across which flow occurs.

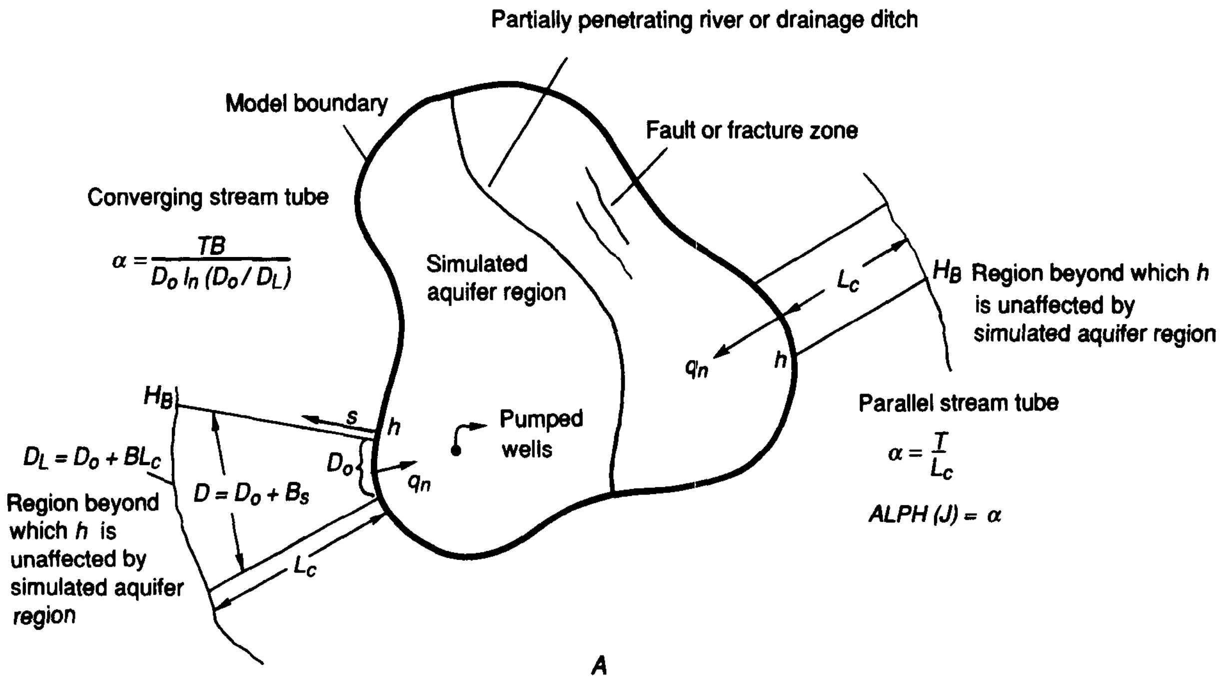
The head-dependent (Cauchy-type) condition can be used to represent ground-water flow across a fault or fracture zone. The flow is analogous to steady vertical leakage except that it occurs along element sides, which are aligned with the trace of the fault or fracture zone in the aquifer (fig. 18), instead of occurring over the area of elements as in vertical leakage. The α term contains hydraulic properties that characterize the material within the fault or fracture zone. These properties are the vertical hydraulic conductivity, K', width, W', and thickness, b', of the fault or fracture zone. An element side can be bounded from above, from below, or from above and below, by the head-dependent (Cauchy-type) condition. The boundary head H_B is the source-layer head or other head external to the aquifer that is associated with providing a boundary flow along the fault or fracture zone.

Values for the boundary or external head H_B can be changed as a function of time during the simulation. Details of this procedure are given in the section "Changing Stresses and Boundary Conditions with Time."

Cross Sections

MODFE can be used to simulate ground-water flow along a cross section defined by a stream line. Cross sections usually are constructed to determine horizontal and vertical components of ground-water flow that are associated with stresses, boundary conditions, and other hydrologic features. Typical applications of cross sections are to simulate the head distribution in the vicinity of a fully or partially penetrating river, infiltration gallery, or similar set of line-oriented stresses (wells) (fig. 19), and to determine the effects of vertical leakage on a system of aquifers and confining beds (fig. 20*A*). Cross-section simulations can be performed on a confined aquifer or on a system of confined aquifers and confining beds in steady or nonsteady state. Because MODFE does not contain the formulation of a moving boundary for the phreatic surface, only steady-state conditions can be simulated for an unconfined aquifer. (For cross sections of unconfined aquifers in steady state, the phreatic surface is a no-flow or specified-head boundary.)

Inputs and program variables in MODFE take on different meanings for cross-section simulations than for simulations in the areal plane. For cross sections, hydraulic conductivity is input for one horizontal direction (x or y) and for the vertical direction as the program variables XTR and YTR, respectively.



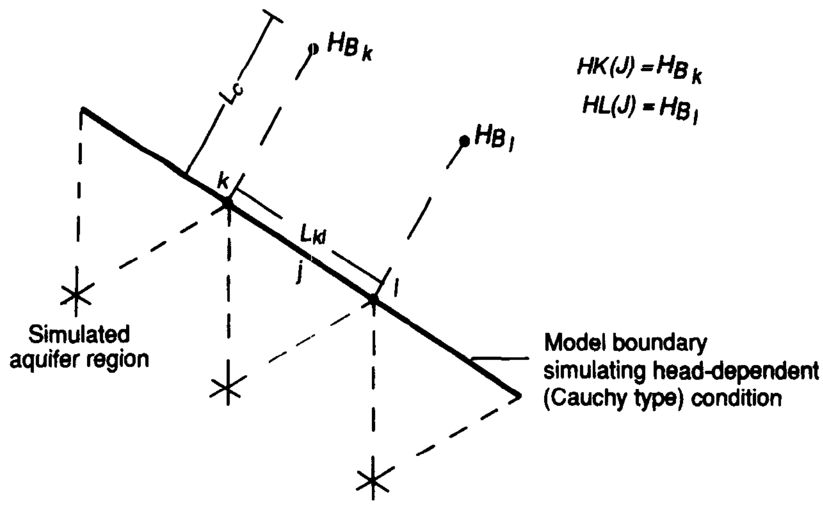


Figure 16.—(*A*) Examples of head-dependent (Cauchy-type) flux q_n across simulated aquifer boundaries; and (*B*) subdivision of boundary using element sides and nomenclature used in MODular Finite-Element model (MODFE).

These program variables usually represent aquifer transmissivity in the x and y directions, respectively, for areal simulations. Specific storage [length^{-1}] is input as the program variable STR for cross sections instead of the aquifer storage coefficient [dimensionless] for areal simulations.

Because the finite-element mesh is oriented in the vertical plane, boundary conditions and stresses that

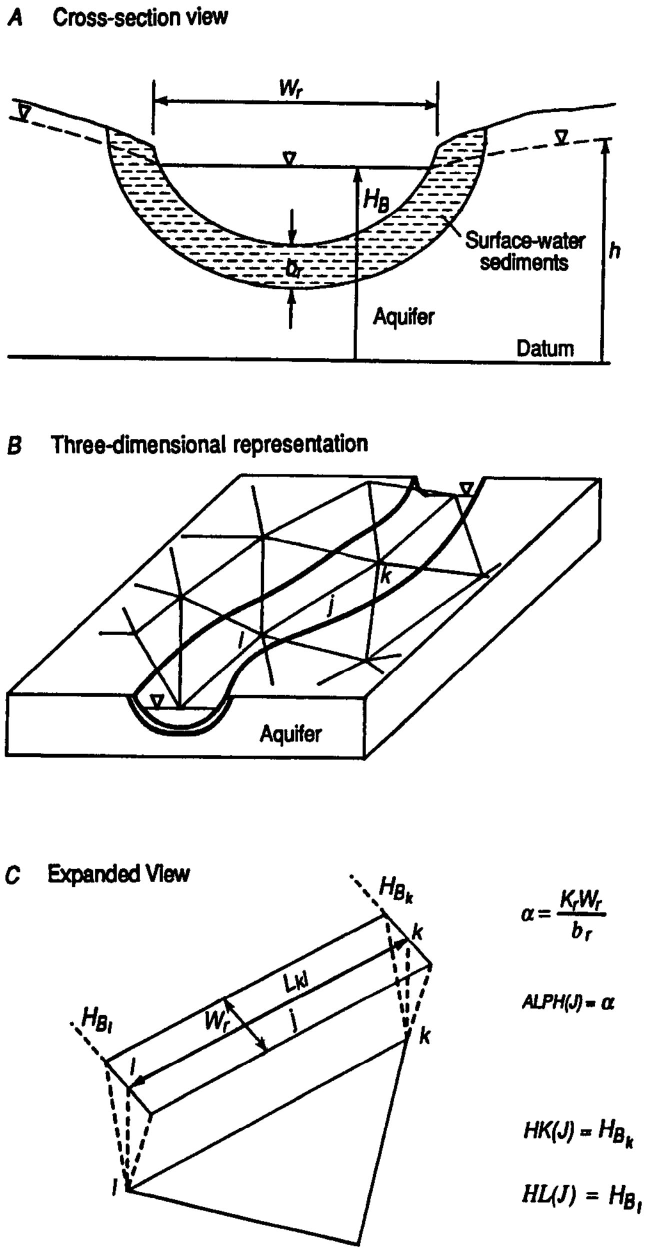


Figure 17.—Rivers represented as head-dependent (Cauchy-type) boundaries; (*A*) cross-section view of river and aquifer; (*B*) three-dimensional representaion of river and aquifer, partially subdivided with finite elements; and (*C*) plan view of element on boundary and nomenclature used in MODular Finite-Element model (MODFE).

are applied to aquifer problems in the areal directions are represented differently in cross section. Stresses and boundary conditions that are represented by points or lines in the areal plane are extended vertically in the cross section and become, respectively, line or areal stresses and boundary conditions (fig. 19). Areal hydrologic features such as vertical leakage and areally distributed recharge are represented as line features in cross-section simulations (fig. 20*A*,*B*). Regional flow across model boundaries is represented as flow across element sides in a manner similar to the representation of regional flow in the areal plane, except the element side is oriented vertically in the plane of the cross section instead of horizontally (fig. 20*C*). In cross section, these hydrologic features are represented as Cauchy-type boundaries, either as specified-flux or as head-dependent (Cauchy-type) flux conditions.

Applications of Cauchy-type boundaries for cross-section simulations are described here using terms that were defined in the sections "Specified Flux" and "Head-Dependent (Cauchy-Type) Flux." The user is referred to those sections for details about Cauchy-type boundaries.

A unit thickness ($b=1$) is assumed to exist normal to the plane of the cross section (figs. 19 and 20). This value is used to complete the formulation of flow across an element side by providing an area across which boundary fluxes and stresses can occur. The unit thickness is included in the following discussions, although it is excluded from computations in MODFE.

Ground-water flow to a fully or partially penetrating river can be simulated in cross section by using the head-dependent (Cauchy-type) condition (fig. 19*A*). The boundary flow is controlled by the head difference ($H_B - h$) and the hydraulic properties of the riverbed sediments. The α term is given as $K_r b/b_r$, where K_r and b_r are the hydraulic conductivity and thickness, respectively, of the riverbed sediments, and b is the unit thickness of the cross section. The river stage is represented as the boundary or external head H_B.

Constant flow to a line of wells or from an infiltration gallery can be represented in cross section by using the specified-flux condition (fig. 19*B*). The volumetric discharge (or recharge) rate, Q_B, is divided by the length of the screened interval or open hole in contact with the aquifer, L, and the distance between wells, L_w, to obtain a unit discharge, q_B, for the specified-flux condition. The unit discharge represents the volumetric flow rate per unit thickness of the cross section, per unit length of screened interval or open hole. For multiple aquifers or for nonhomogeneous conditions, Q_B is proportioned according to the flow rate, q_{Bi}, and length, L_{zi}, of each zone i that contributes or receives water across the boundary (fig. 21). Thus, different discharge or recharge rates can be represented by boundary-condition zones having different values of q_{Bi}. The sum of the flow rates q_{Bi} over all boundary-condition zones gives the value Q_B/L for the multiaquifer or nonhomogeneous system.

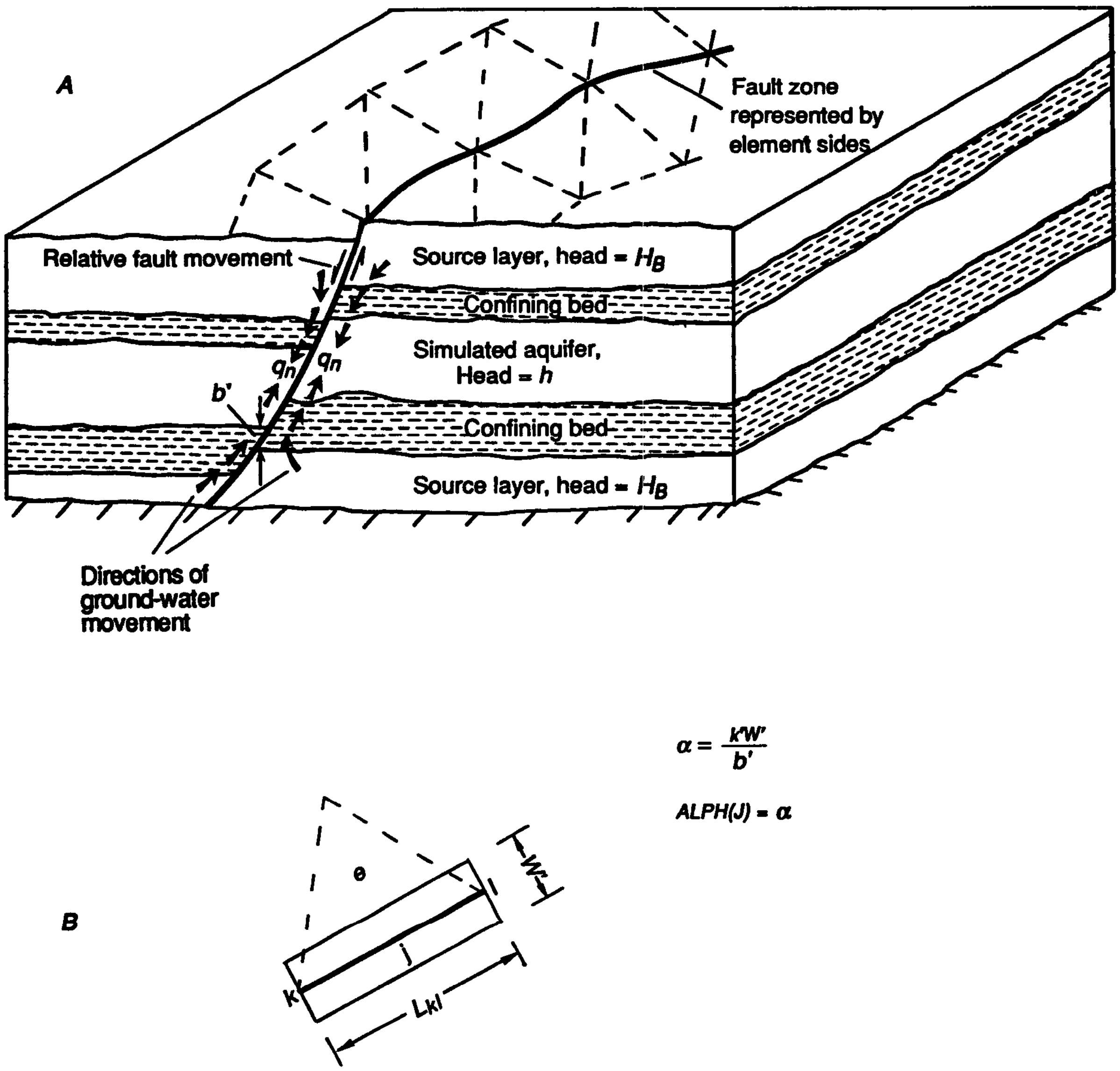
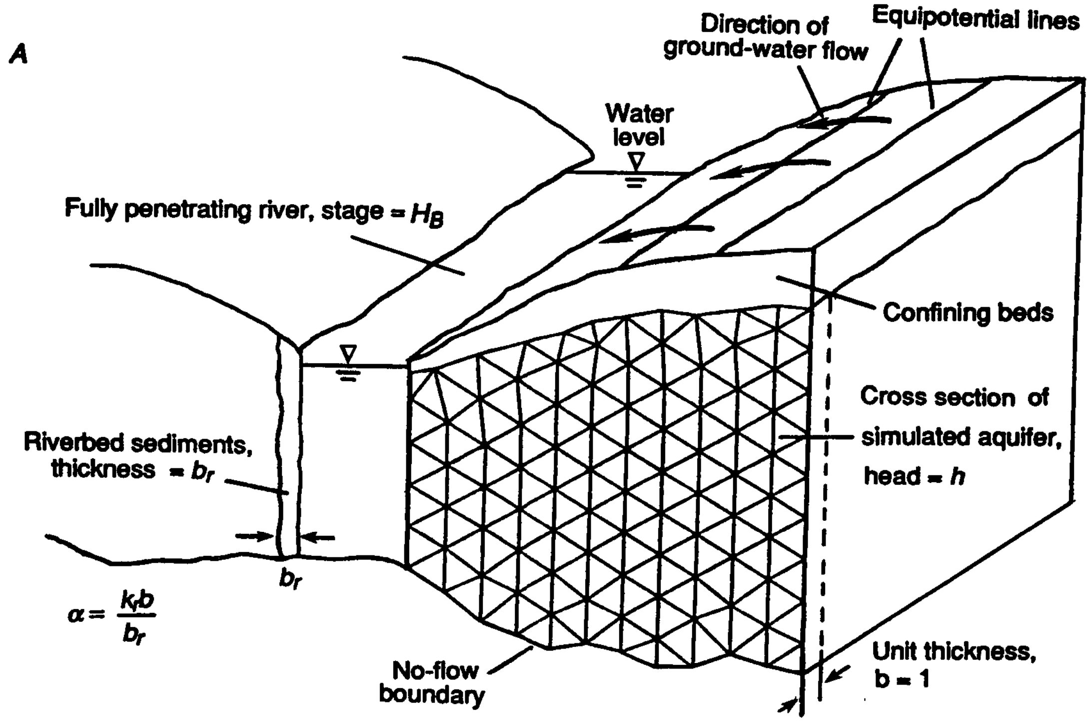


Figure 18.—Ground-water flow along fault zone represented as head-dependent (Cauchy-type) boundaries; (*A*) three-dimensional representation of simulated aquifer, partially subdivided with finite elements with ground-water flow along fault zone; and (*B*) plan view of element side on boundary.

Steady-vertical leakage is simulated in cross section as a head-dependent (Cauchy-type) boundary (fig. 20*A*). The external or boundary head H_B for this application of the boundary condition represents the source-layer head. The α term is computed as the product of vertical hydraulic conductance (vertical hydraulic conductivity divided by thickness) and unit thickness normal to the plane of the cross section. This computation is given as $K'b/L_c$ in figure 20*A*, where K' is the vertical hydraulic conductivity of the material between the aquifer and the source-layer head, H_B, b is the unit thickness of the cross section, and L_c is the distance from the boundary to the source-layer head H_B. Usually, K' is the vertical hydraulic conductivity of a confining bed and L_c is confining-bed thickness.

Regional flow across model boundaries is represented in cross-section by head-dependent (Cauchy-type) flux conditions (fig. 20*C*). The α term is computed as Kb/L_c, where K is the horizontal hydraulic conductivity (either the x or y direction in the plane of the cross section) of the material in the region between boundary side j and the external head H_B; b is the unit thickness of the cross section, and L_c is the distance from side j to H_B.



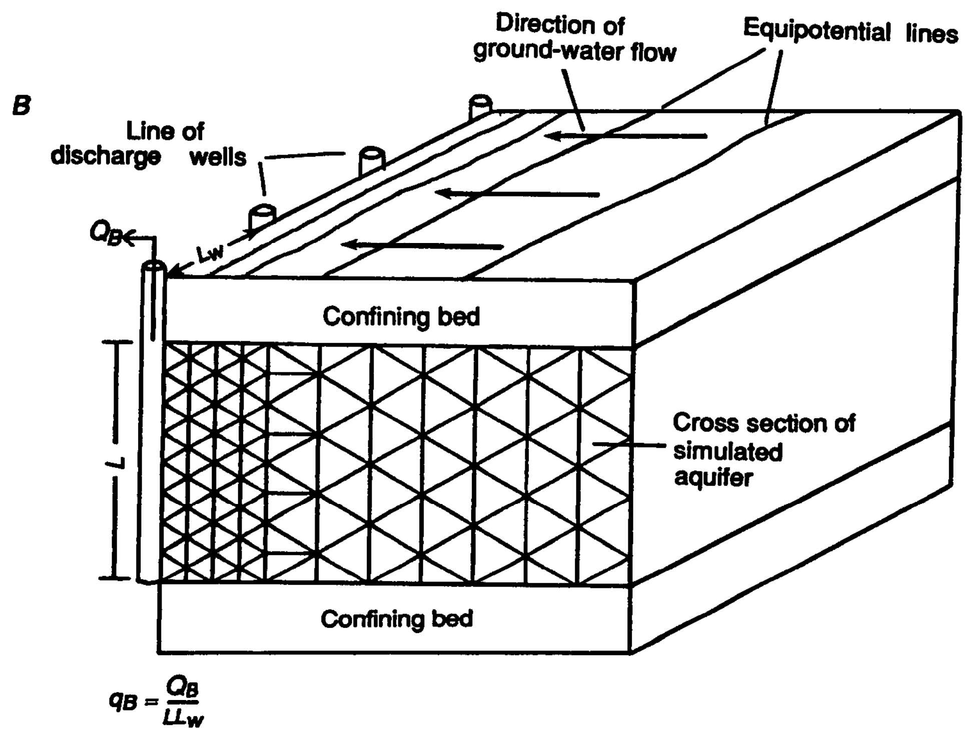


Figure 19.—Cauchy-type boundaries used in cross section to represent (*A*) fully penetrating river; and (*B*) line of discharge wells having volumetric flow rate Q_B.

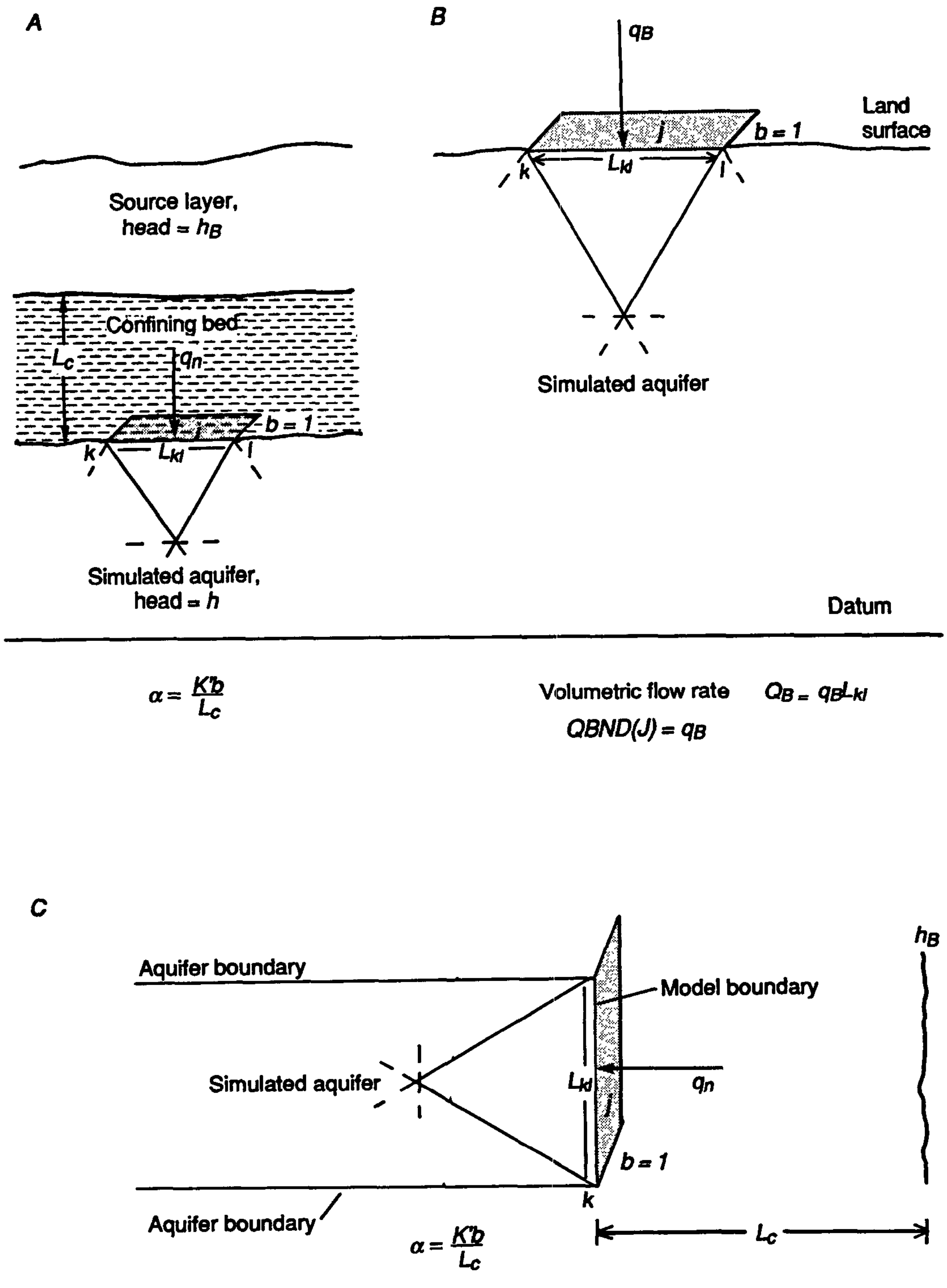


Figure 20.—Cauchy-type boundaries in aquifer cross sections for simulating (*A*) steady-vertical leakage; (*B*) areally distributed stresses; and (*C*) lateral flow across model boundaries.

Areally distributed stresses are represented in cross section as specified-flux boundaries along element sides (fig. 20*B*). The unit rate, W, in equation (1) is multiplied by the unit thickness of the cross section, b, to obtain the flow rate, q_B, per unit length across the element side. The volumetric flow rate, Q_B, is computed in MODFE by multiplying q_B by the length of the boundary side, L_{kl}.

As in the areal application of Cauchy-type boundaries, element sides that contain the same values of q_B

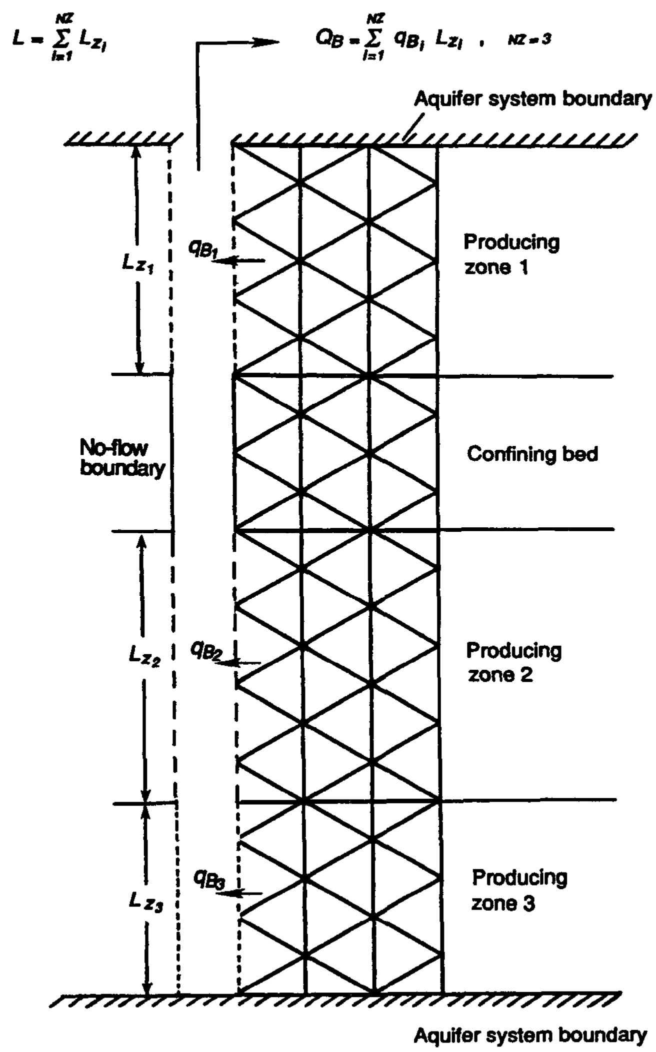


Figure 21.—Cross section of nonhomogeneous or multi-aquifer conditions showing flow to one well in the line of wells shown in figure 19*B* and application of specified-flux boundaries.

and α can be grouped together into one boundary-condition zone. Also, time variance of the boundary head H_B and of the unit flow rate q_B are permitted in cross-section simulations. Details of these features are given in the sections "Hydraulic-Property and Boundary-Condition Zones" and "Changing Stresses and Boundary Conditions With Time."

Axisymmetric Flow

An extension to cross-section simulations is the ability of MODFE to solve flow problems in axisymmetric-cylindrical (or radial) coordinates. A typical application is for simulating flow to a well, either in a single aquifer or in a layered system of aquifers and confining beds. As in cross-section simulations, transient-state, unconfined conditions involving a moving phreatic surface cannot be represented by axisymmetric flow with MODFE.

For axisymmetric-flow problems, the x axis is replaced by r, and y by z (fig. 22). The finite-element mesh is rotated about the z axis (r=0) so that each element represents a ring-like volume of material having a triangular cross section in the r-z plane (see figure 17 in Cooley, 1992). This rotation is not performed explicitly in MODFE; however, symmetry of the aquifer domain about the z axis is assumed. The concept of rotating the r-z plane about the z axis is necessary for identifying radial symmetry in field problems and for subsequent application of MODFE. The flow problem that is solved by MODFE is equivalent to a slice of aquifer material having a thickness of one radian, and is obtained by dividing the resulting equations by 2π (see following sections).

Axisymmetric flow implies that the aquifer material and boundary conditions are identical along any radius drawn from the z axis. Symmetry is assumed for hydraulic properties and boundary conditions about the line r=0, which is the z axis (fig. 22). For flow to a well, the line of symmetry is along the center of the well bore. Thus, element sides on the model boundary that is closest and parallel to the z axis, at $r=r_w$, trace the aquifer material in contact with the well bore. Similarly, zones for hydrologic properties and element sides representing boundary conditions, such as at $r=r_B$, are assumed to be rotated about the z axis. Because this rotation can produce many different shapes of hydrologic boundaries, care must be taken when designing the finite-element mesh for axisymmetric-flow as undesired boundary shapes can be formed.

Inputs for axisymmetric flow are analogous to those required for cross-sectional problems. The program variables XG and YG are used to represent radial (r) and vertical (z) node coordinates, respectively. The program variables XTR and YTR are used to input hydraulic conductivity in the radial and vertical directions, respectively, and specific storage is input as the program variable STR. The axisymmetric computations are invoked in MODFE by inputting a value of one (1) for the indicator variable IRAD.

Flow to a well and boundary conditions in axisymmetric flow can be represented in MODFE by using Cauchy-type boundary conditions. Applications of these boundary conditions to axisymmetric flow are given in the following sections. Details about these conditions are given in the sections "Specified Flux" and "Head-Dependent (Cauchy-Type) Flux."

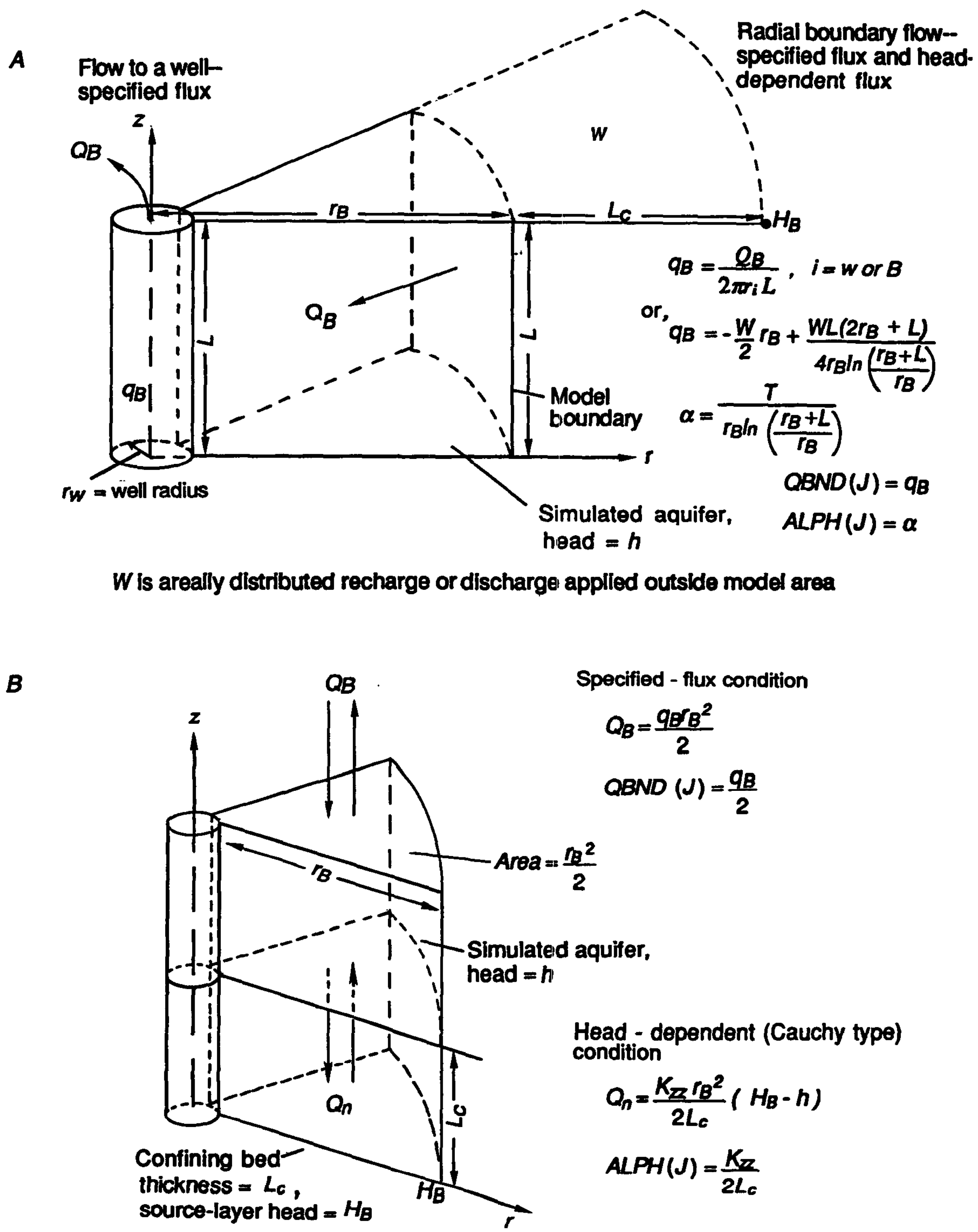


Figure 22.—Examples of Cauchy-type boundaries (specified flux and head-dependent flux) in axisymmetric flow for simulating boundary flow (*A*) in radial direction; and (*B*) in vertical direction.

Flow to a Well

Flow to a well in axisymmetric coordinates is represented in MODFE as a specified-flux boundary (fig. 22*A*). The specific discharge q_B [length/time] of the well is represented by the unit flow rate for the boundary condition, and is computed as

$$q_B=\frac{Q_B}{2\pi r_w L} \tag{4}$$

where Q_B is the volumetric flow rate [length^3/time], r_w is the well radius [length], and L is the length of screen or open hole to the aquifer. A nonzero well radius is required by MODFE; hence, the boundary length L cannot be located at the z axis. The denominator of equation (4) represents the cylindrical area of the boundary that results from rotation about the z axis. The 2π is factored out of Q_B because computations in MODFE represent a slice of the aquifer problem that is one radian thick.

As described previously for cross sections, the volumetric flow rate Q_B can be derived from one boundary zone of length L or from several boundary zones L_{zi} totalling L (fig. 21). In addition, each boundary zone can have a different unit flow rate q_{Bi}, corresponding to different producing zones in a nonhomogeneous aquifer or a multiaquifer system. These flow rates are computed for boundary zone i according to equation (4) by replacing q_B by q_{Bi}, L by L_{zi} and Q_B by Q_{Bi}, where Q_{Bi} is the known volumetric flow rate for the zone. Values of q_B or q_{Bi} are represented in MODFE as the program variable QBND(J) for each side j on the boundary.

Boundary Conditions Parallel to Z Axis

Boundary conditions that are parallel to and at a distance from the z axis in axisymmetric flow can be simulated by using Cauchy-type boundary conditions (fig. 22*A*). For a specified-flux boundary situated at a distance $r=r_B$ from the z axis, the unit flow rate q_B is computed in the same manner as for flow to a well by replacing r_w by r_B in equation (4). The term L in equation (4) is the length of the boundary across which the specified flux occurs, and Q_B is the volumetric flow rate. Care should be taken when applying this boundary condition as the specified flux is assumed to occur across the cylindrical area $2\pi r_B L$ that results from rotating r_B about the z axis.

A head-dependent (Cauchy-type) flux parallel to the z axis can be used to simulate flow across the outer boundary of the model area that results from stresses applied within the model area. This boundary condition usually supplies water to the model area in response to a pumped well located at $r=r_w$. Boundaries located at a distance r_B from the z axis are rotated in a manner similar to that of specified-flux boundaries and become boundaries having the cylindrical area $2\pi r_B L$, where L is the length of the boundary (fig. 22*A*). The unit flow rate normal to this cylindrical area is given by q_n, which is defined as in Cartesian coordinates by the product of the term α and the head difference $H_B - h$. The term α is computed as K_{rr}/L_c, where K_{rr} is the hydraulic conductivity of the material between the boundary and the external or boundary head, H_B, and L_c is the distance from the boundary to H_B.

Boundary Conditions Parallel to R Axis

Vertical flow across model boundaries parallel to the r axis in axisymmetric flow can be represented with Cauchy-type conditions. These boundary conditions represent areally distributed flow rates across the upper and lower (vertical) boundaries of the r-z plane because this plane is rotated about the z axis (fig. 22*B*). The specified-flux condition can be used to represent areally distributed stresses, such as applied recharge, constant rates of precipitation, or evapotranspiration. The head-dependent (Cauchy-type) condition can be used to represent steady vertical leakage or regional flow.

Because the axisymmetric-flow problem is represented in MODFE as a slice of aquifer material that is one radian thick, computations and data inputs for boundary conditions are affected by the factoring of 2π out of the model equations. For example, a vertical boundary of length r_B parallel to the r axis permits flow across the area of a circle of radius r_B, or πr_B^2. However, the area of one radian of arc (radius r_B) is $(\pi r_B^2)/2\pi$, or, $(r_B^2)/2$ (fig. 22*B*). Thus, the volumetric flow rate Q_B across a specified-flux boundary of length r_B is represented in MODFE as the flow across the area of one radian, or

$$Q_B=\frac{(q_B)r_B^2}{2} \tag{5}$$

where q_B is the unit recharge or discharge rate, which is identical to the W term of equation (1).

Values of $q_B/2$ are input to MODFE as the program variable QBND(J) for element side j along r_B. Most likely, more than one element side will be needed to represent r_B. For each side j on the boundary, Q_B is distributed proportionately according to the area that is created by rotating the side through one radian of arc.

For a head-dependent (Cauchy-type) boundary of length r_B, the volumetric flow rate across the area $r_B^2/2$ is given as

$$Q_n = \frac{(K_{zz}) r_B^{\,2}}{2L_c} (H_B - h) \qquad (6)$$

where K_{zz} is the vertical hydraulic conductivity of the material between the boundary r_B and the external head H_B located at a distance L_c from the boundary (fig. 22*B*). Values of $K_{zz}/2L_c$ are input to MODFE as ALPH(J) for boundary side j on r_B.

Although values for ALPH(J) and QBND(J) can vary for element sides on a Cauchy-type boundary, symmetry of the axisymmetric-flow problem may prevent different values for these boundary terms from being used. Boundary-condition zones represent concentric-circular areas about the z axis, and each zone can be assigned distinct values for ALPH(J) or QBND(J). However, the geometric configuration of concentric rings of aquifer material, each containing different values for Cauchy-type boundaries, may not represent the true distribution of boundary conditions for the aquifer problem. Therefore, boundary-condition zones should be selected carefully for axisymmetric-flow problems.

Water-table (Unconfined) Conditions

Two-dimensional ground-water flow in a water-table (unconfined) aquifer that is assumed to be governed by the nonlinear form of equation (1) can be simulated by MODFE. The flow is nonlinear because the effective aquifer transmissivity is a function of the saturated aquifer thickness, which changes as the hydraulic head changes during the simulation. The aquifer-storage characteristics also may be nonlinear if, during the simulation, the aquifer converts from confined to unconfined conditions or from unconfined to confined conditions. However, for water-table conditions without conversion, the aquifer-storage properties are assumed to be constant in time and are defined by the specific yield. Changes in aquifer-storage properties are discussed in the section "Conversion Between Confined and Unconfined Aquifer Conditions."

The variation in transmissivity with time is represented in MODFE by a predictor-corrector technique that approximates hydraulic head at the advanced time level. Details of this technique are given in the section "Unconfined Flow," in Cooley (1992). In general, head changes that are computed during the predictor step are used to obtain estimates of aquifer thickness and transmissivity for the corrector step. The corrector step solves the finite-element equations for head changes over the same time level as the predictor step. Head changes during the corrector step are caused by using different transmissivity values from those used during the predictor step. Updates to aquifer thickness and transmissivity for the advanced time level are based on head changes from both steps.

The ability of the predictor-corrector technique to approximate the time variance in aquifer thickness and transmissivity is related to the size of the time-steps used to subdivide the simulation period. Inappropriately large time steps usually increase the errors associated with approximating thickness and transmissivity during the simulation. Approximation errors are manifested in model results as large flow imbalances in the water-balance summary and as incorrect values of computed water levels. However, using time steps that are too large for the nonlinearity imposed on the aquifer problem by water-table conditions may not be the only cause of large flow imbalances and an incorrect solution. Errors associated with improper use of boundary conditions or inaccurate specification of hydraulic properties also contribute to inaccuracies in the model results, and determining the cause of errors in model results may be a difficult task. The effects of the time-step size on the approximation of aquifer thickness and transmissivity and on the flow imbalance are discussed in the sections "Water-table (Unconfined) Conditions" and "Water-Balance Summary and Flow Imbalance" in Torak (1993).

To solve water-table problems by using MODFE, the user must structure the main program to contain subroutines that perform the water-table computations and the predictor-corrector technique. Details on structuring MODFE and diagrams showing program structures for steady-state and nonsteady-state water-table versions are given in the section "Program Structures and Lists of Main Programs" in Torak (1993). Programming details of the water-table computations are given in the section "Water-table (Unconfined) Conditions" in Torak (1993).

Inputs to MODFE for water-table simulations consist of values for hydraulic conductivity, altitude of the top of the aquifer, aquifer thickness, and specific yield (fig. 23). Hydraulic conductivity is represented with program variables XTR and YTR for the x and y directions, respectively, and is input by hydraulic-property zone. Aquifer thickness and the altitude of the aquifer top are input by node and are represented, respectively, by program variables THK and TOP. Nodal values for TOP are selected as either the altitude of land surface or the altitude of the base of an overlying confining bed. Specific yield is represented by program variable SY and is input by aquifer-property zone. Aquifer storage coefficient also is required as input for simulations involving conversion between confined and unconfined conditions. The storage coefficient is represented as program variable

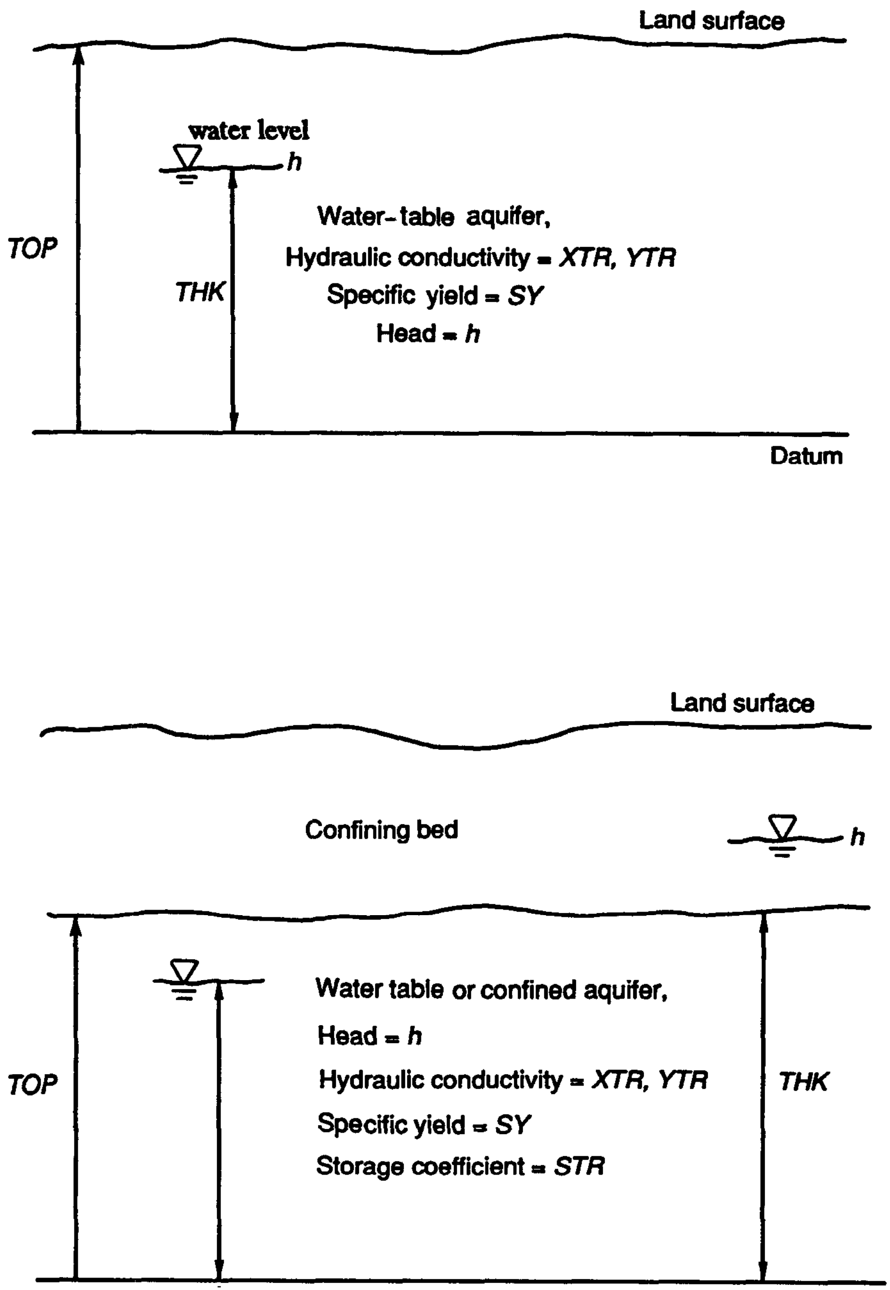
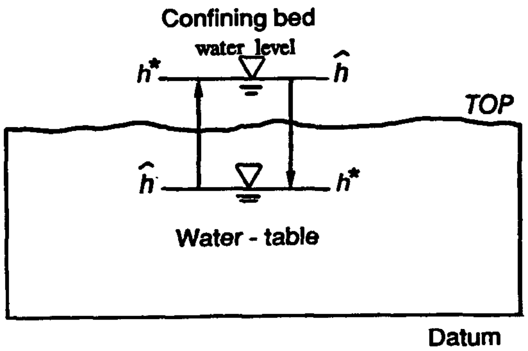


Figure 23.—Water-table aquifer conditions and nomenclature used in MODular Finite-Element model (MODFE).

STR. Details about data inputs are contained in the section "Input Instructions."

Simulations that cause inundation of land surface by rising ground-water levels from a surficial aquifer also will cause conversion from unconfined to confined conditions. This is because the nonsteady-state water-table versions of MODFE formulate aquifer conversions automatically regardless of whether or not an overlying confining bed is present. Conversion (and inundation) occurs for a surficial aquifer when the aquifer head exceeds the value of TOP at a node. For this case, the value of TOP represents land surface altitude, which is compared with aquifer head, H, to update thickness and change aquifer-storage terms at each node. Although inundation may not be the desired simulation result, conversion of storage terms can be avoided when inundation occurs by setting values of aquifer storage coefficient, program variable STR, equal to the specific yield, SY, on input, for water-table simulations. If inundation of land surface



$\hat{h}$ is aquifer head at beginning of time step, h^* is predicted aquifer head at end of time step

Figure 24.—Configuration of aquifer head and altitude of base of overlying confining bed (*TOP*) for conversion between confined and unconfined conditions.

is the intended result, then the value input for STR would be somewhat larger than the value of specific yield. However, use of MODFE for inundation-type problems should proceed with care because the storage-conversion formulation in MODFE was not designed for this application, and overland flow is not simulated for the inundation condition.

Conversion Between Confined and Unconfined Aquifer Conditions

For certain ground-water-flow problems, the aquifer head may be above the altitude of the base of an overlying confining bed (confined conditions) at one instant in time, only to drop below the base of the overlying confining bed at another instant in time (unconfined conditions). Conversely, the aquifer head may be below the altitude of the base of an overlying confining bed at one time, only to increase above the base of the overlying confining layer at later time (fig. 24). These configurations of aquifer head and altitude of the base of an overlying confining bed describe the conversion of an aquifer between confined and unconfined conditions. Conversion causes changes in the manner in which water is released or accumulated by aquifer storage as the confined storage coefficient is smaller than the specific yield of a water-table aquifer by at least two or three orders of magnitude. Computations are performed in MODFE to account for the effects on computed hydraulic head of changing aquifer storage properties during conversion.

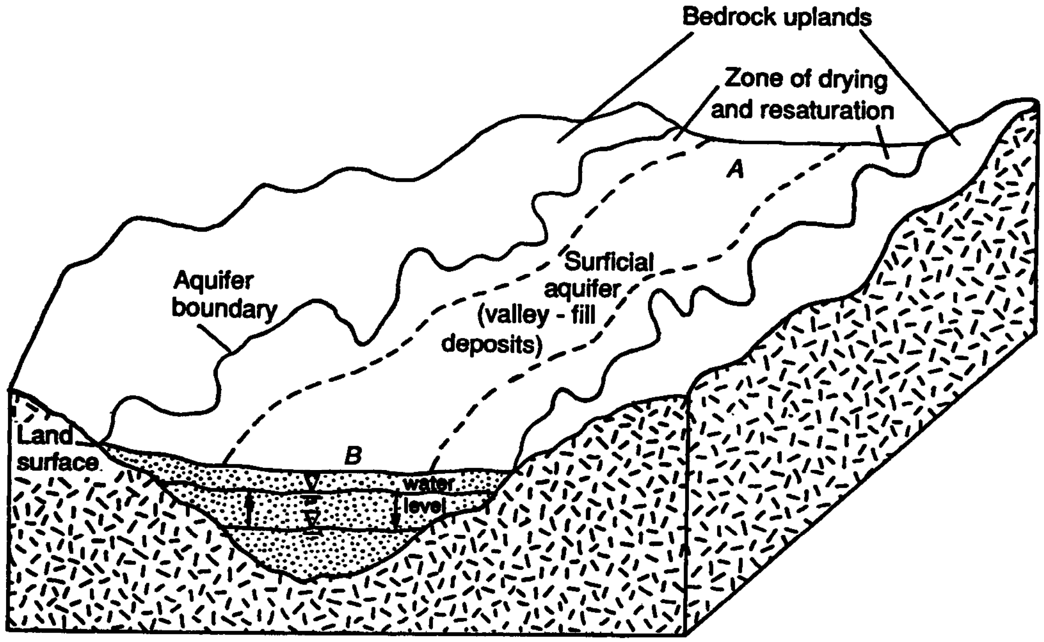
Computations that allow aquifer conversion between confined and unconfined conditions are performed automatically in the nonsteady-state water-table versions of MODFE. Details of structuring the main program for water-table simulations are given in the section "Program Structures and Lists of Main Programs" in Torak (1993). The nonsteady-state water-table versions of MODFE use the predictor-corrector technique to approximate aquifer thickness and transmissivity during the simulation. Checks are made on the hydraulic head at each node at the beginning of a time step and at the end of the predictor step to determine whether aquifer conversion had taken place (fig. 24). Aquifer conversion occurs when the head at a node rises above or drops below the altitude of the base of an overlying confining bed during a time step (represented as TOP in fig. 24). Details of the computations of aquifer thickness and storage at nodes that experience aquifer conversion are given in the section "Conversion Between Confined- and Unconfined-Aquifer Conditions" in Torak (1993).

The inputs required to simulate conversion between confined and unconfined aquifer conditions are the same as those described for water-table simulations (see previous section and fig. 23.). Values for the confined storage coefficient and the specific yield are input for aquifer-conversion problems. The storage coefficient is represented in MODFE as the program variable STR, and the specific yield is represented as the program variable SY. The altitude of the base of the overlying confining bed is input as the variable TOP. Details of inputs for water-table simulations and for aquifer conversions are given in the section "Input Instructions."

Drying and Resaturation of Aquifer Material

A water-table aquifer may respond to stresses and boundary conditions by drying (desaturating) along its external boundaries, such as a surficial aquifer of valley-fill deposits in contact with bedrock-valley walls (fig. 25), or by localized drying within the aquifer region, such as in the vicinity of a pumped well. Conversely, stresses and boundary conditions may be such that dry aquifer material may become saturated, or resaturated, after a period of desaturation (dryness). Conditions of drying and resaturating parts of a water-table aquifer are simulated automatically in the water-table versions MODFE. The drying condition is represented in MODFE as a zero or negative value of aquifer thickness (program variable THK) at a node.

Negative values of aquifer thickness enable the water-table to pull away from the dry node and the element to become partially dewatered (fig. 26). Because the head at the dry node is allowed to drop below the base of the aquifer, hydraulic gradients within the partially dry element and within neighbor-



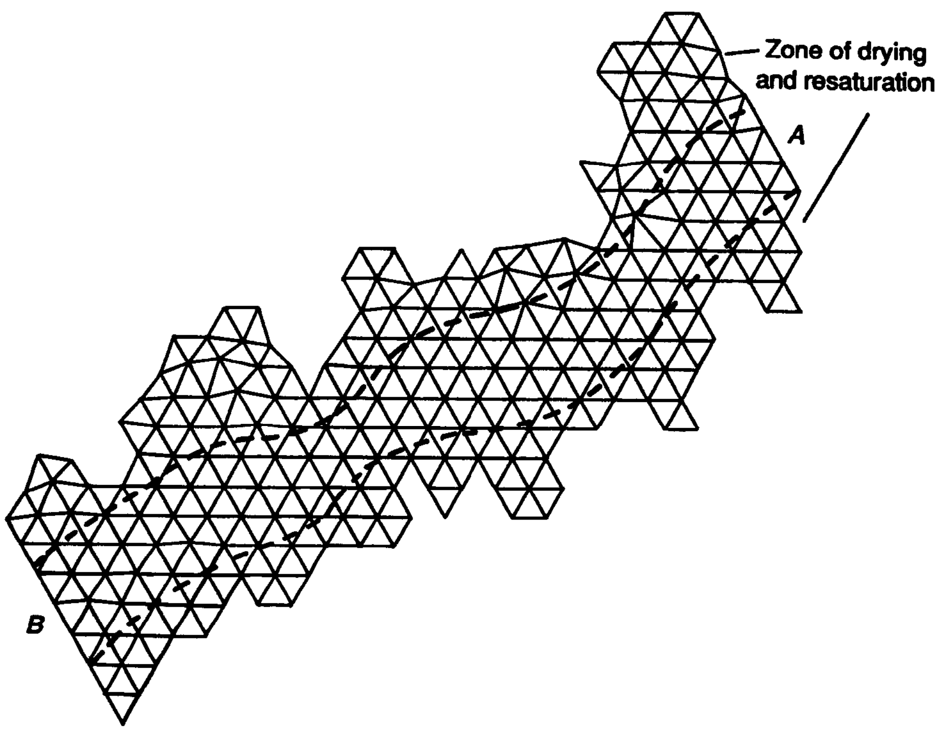


Figure 25.—(*A*) Block diagram of surficial aquifer (valley-fill deposits) bounded by bedrock uplands; and (*B*) finite-element mesh used to simulate drying and resturation of aquifer material.

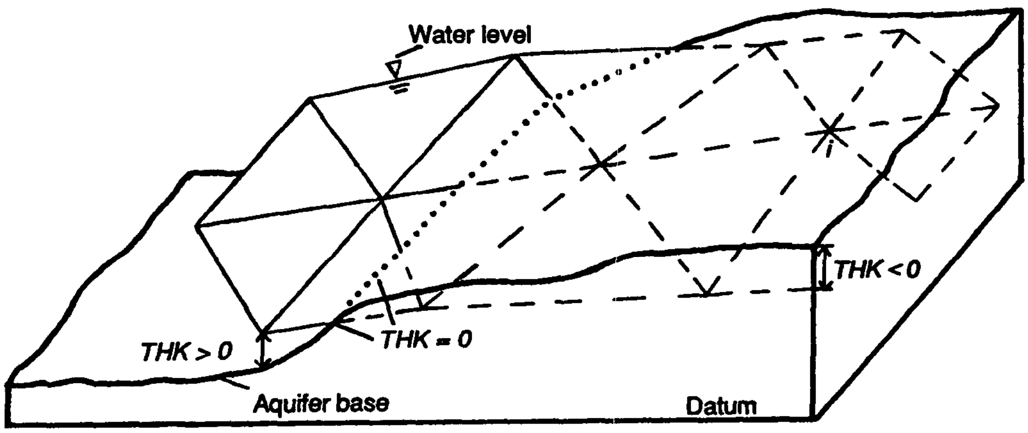


Figure 26.—Potentiometric surface of water-table aquifer, subdivided by finite-elements, showing drying conditions.

ing elements can remain nearly horizontal during the drying process. This configuration of aquifer head may represent a more realistic hydrologic condition than if head at the dry node were fixed at the base of the aquifer during drying. Negative thickness values at dry nodes are not used in computations; however, they are used to determine the amount of head increase that is needed to resaturate the aquifer at the dry node.

A dry node continues to be part of the flow system, and head changes continue to be computed there, as long as the node is linked to nondry nodes or as stresses at the dry node indicate recharge conditions. Resaturation occurs when boundary conditions, lateral flow, or stresses allow water levels to increase to the point where head changes at the dry node cause a positive value of aquifer thickness to be computed.

Head changes are not computed at dry nodes that are completely surrounded by other dry nodes, thus creating a patch of dry elements (fig. 26). For this condition, resaturation can occur only if a node that is linked to the center of a patch of dry elements becomes resaturated. Thus, the dry node in the center of the patch becomes linked to nondry nodes in the model area and can resaturate. Details of the equation development for dry nodes are given in the section "Drying and Resaturation of Nodes" in Cooley (1992). Programming details are given in the section "Aquifer Drying and Resaturation" in Torak (1993).

Drying and resaturation are nonlinear processes that require the predictor-corrector technique to update aquifer thickness during a time step. To assist the user in keeping track of dry nodes during simulation, the following message is printed out from the corrector step:

NODE ___ PREDICTED TO BE DRY
PREDICTED AQUIFER THICKNESS AT NODE =___
NET FLOW AT NODE =___

where the appropriate values for the node number, aquifer thickness, and sum of known flows at the dry node are printed in the blank spaces. If the net flow at a dry node is negative, indicating discharge, then, obviously, ground water cannot be extracted from the aquifer at that rate. In an attempt to simulate a more realistic aquifer problem by keeping the node from going dry, the discharge rate is decreased by half of its current value and a message is printed out stating that this has occurred. The decreased discharge rate is used at the dry node for the corrector step and for subsequent time steps or iteration levels, even if the node remains saturated. If the node is predicted to go dry on the time step or iteration level following a decrease in discharge, then the discharge rate is decreased again by half of its current value. The decrease in discharge rate by half is completely arbitrary. The printed message alerts the user of the dry node so that this condition can be evaluated after the simulation. The user can consult the programming details in the section "Aquifer Drying and Resaturation" in Torak (1993) to modify the program as needed to provide other consequences of drying at a node containing a net discharge rate, such as decreasing the discharge rate by an amount other than one half, not decreasing the discharge rate, or stopping MODFE.

Nonlinear Head-Dependent Flux

Boundary conditions containing head-dependent (Cauchy-type) fluxes and steady vertical leakage usually are linear; that is, the volumetric flow rate is proportional to a head difference, and the mathematical expression defining the boundary condition is fixed for all values of aquifer head (see sections "Head-Dependent (Cauchy-Type) Flux" and "Steady Vertical Leakage"). However, for some aquifer problems, the flow rate from these conditions can be limited to a maximum or minimum value depending on the position of the aquifer head relative to a controlling head or altitude. These limitations require that the mathematical expression for the boundary condition change depending on evaluation of the aquifer head with the controlling head or altitude. Because the form of the mathematical expression defining the boundary condition can change as the aquifer head changes, the boundary condition and the functions used to describe them are nonlinear. Some examples of nonlinear head-dependent fluxes are: (1) flow across a riverbed when the water-table drops below the altitude of the bottom of the riverbed sediments, (2) flow to or from an overlying fault or fracture zone or an overlying source layer (steady vertical leakage) when the aquifer converts between confined and unconfined conditions, (3) spring discharge or irrigation drainage, and (4) evapotranspiration.

The following sections describe the application of nonlinear head-dependent functions for simulating nonlinear boundary conditions such as those listed above. The mathematical representation of nonlinear boundary conditions in MODFE permits many more applications than those just given. Consequently, the nonlinear head-dependent functions are categorized as Cauchy type, point sinks, and steady vertical leakage instead of as rivers, springs, and evapotranspiration, respectively.

Simulation of nonlinear head-dependent flux requires use of the nonlinear versions of MODFE, which contain either the predictor-corrector technique for nonsteady-state conditions or an iterative method for steady-state conditions. The computations for nonlinear head-dependent (Cauchy-type) functions and nonlinear point sinks are contained in a set of subroutines that begin with the letters GN. These subroutines and their call statements are added to the program structure by the user if these simulation capabilities are needed. The computations for nonlinear steady vertical leakage are contained in a similar set of subroutines beginning with the letters VN. Instructions for incorporating nonlinear head-dependent functions into MODFE are given in the section "Program Structures and Lists of Main Programs" in Torak (1993).

Cauchy Type

The nonlinear form of the head-dependent Cauchy-type boundary can be used to compute flow rates across line-oriented boundaries that are limited to a maximum or minimum value by a controlling head or altitude. Examples of hydrologic features that can be simulated by using this boundary condition are similar to those given for the linear case, and include flow across a riverbed, fault or fracture zone, or a model boundary (see figs. 16–18, and 20). For simulating flow across a riverbed, the flux is limited to a maximum inflow rate to the aquifer when the aquifer head h drops below the altitude of the bottom of the riverbed sediments z_r (fig. 27*A*). The flux across a fault or fracture zone that is situated in a confining bed overlying an aquifer is limited to a maximum inflow rate when the aquifer undergoes conversion from confined to unconfined conditions (fig. 27*B*). Flow to an irrigation drain or slough can be limited so that only discharge from the aquifer occurs when the aquifer head rises above the altitude of the feature (fig. 27*C*). Other applications in areal or cross-sectional dimensions may limit the flux to a maximum rate depending on aquifer geometry or the flow problem to be solved.

The expression for the flux across a nonlinear head-dependent (Cauchy-type) boundary is given by equation (152) in Cooley (1992). The head or altitude that limits the flow rate is represented by the term z_r. The boundary or external head is given by H_r and is analogous to H_B in equation (6) for a (linear) head-dependent (Cauchy-type) boundary. The term α_r is analogous to the α term in equation (6) (see the discussions of α and H_B in the section "Head-Dependent (Cauchy-Type) Flux" for descriptions of these terms).

Definitions of H_r and z_r vary depending on the application of the nonlinear boundary condition. For simulating flow across a riverbed, H_r is the altitude of the river stage and z_r is the altitude of the bottom of the riverbed sediments (fig. 27*A*). To simulate flow through a fault or fracture zone, z_r is the altitude of the base of the zone in the overlying confining bed, and H_r is the source-layer head or other head that governs the flow rate through the zone (fig. 27*B*). To simulate a discharge-only boundary, such as a drain, z_r is the altitude of the drain and H_r is set equal to z_r (fig. 27*C*).

Like H_r and z_r, the term α_r is defined according to the particular application of the nonlinear boundary

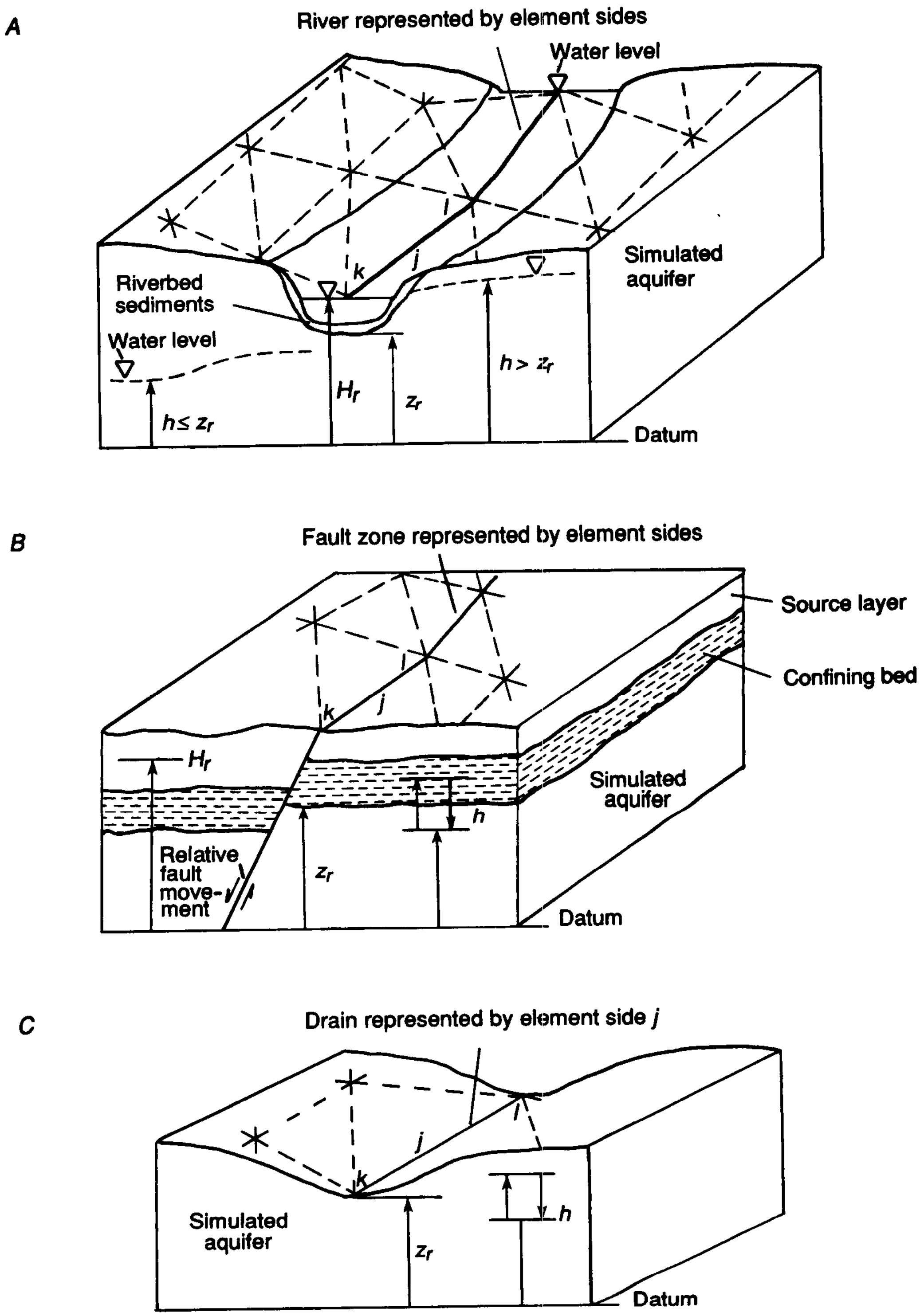


Figure 27.—Nonlinear head-dependent (Cauchy-type) boundaries simulating (*A*) flow across a river bed; (*B*) flow along a fault zone; and (*C*) flow to an irrigation drain.

condition. For simulating a river, fault or fracture zone, or boundary flow, α_r is identical to the α term defined previously for simulating these features by the linear boundary condition (see section "Head-dependent (Cauchy-type) Flux"). For simulating a drain, α_r is assigned values that relate the hydraulic characteristics of the drain (geometry, aperture or diameter of the opening (if applicable), material type, etc.) to the flow rate. Usually, values for α_r are adjusted during calibration until the computed flow rate matches the observed drain discharge.

Inputs for nonlinear head-dependent (Cauchy-type) boundaries consist of nodal values of H_r and z_r, node numbers defining the boundary, and values of α_r for each element side on the boundary. Before inputting these values, the number of boundary sides and zones are input, respectively, as the program variables NBNC and NLCZ. These values are used to allocate computer storage for program variables associated with this boundary condition. The following variables are used to input terms for the nonlinear head-dependent (Cauchy-type) boundaries at nodes k and l, respectively, on boundary side J (fig. 27): node numbers are input as KR(J) and LR(J); boundary or external head H_r is input as HRK(J) and HRL(J); and the controlling head or altitude is input as ZRK(J) and ZRL(J). Additional descriptions of these data inputs are given in the section "Input Instructions."

The α_r term is input by boundary side as the program variable GC(J). Boundary sides that have the same value of α_r can be grouped into one boundary-condition zone for input. Details of establishing boundary-condition zones are given in the section "Hydraulic Property and Boundary-Condition Zones." A time-varying boundary condition can be represented in MODFE by changing values for H_r during the simulation. Typical applications are for changing river stages and source-layer heads with time. Details about changing H_r with time are given in the section "Changing Stresses and Boundary Conditions with Time."

Point Sinks

Hydrologic features that create nonlinear head-dependent discharge at point locations in an aquifer can be represented with the nonlinear head-dependent flux called point sinks. These boundary conditions are represented by nodes in the finite-element mesh. Typical applications of nonlinear point sinks are for representing springs and irrigation drains (fig. 28), where the hydrologic conditions permit only discharge from the aquifer. The flux out of the aquifer is zero if the aquifer head, h, is below the reference altitude z_p; thus, discharge occurs only for cases where h is greater than z_p. For springs, z_p is the altitude of the discharge point in the aquifer (fig. 28*A*). For irrigation drains, z_p is the altitude of the drain opening (fig. 28*B*).

The volumetric flow rate from nonlinear point sinks is expressed in equation (106) in Cooley (1992) as the product of a discharge coefficient, C_{pi}, and the head or altitude difference $(z_{pi} - h_i)$, for node i. The term C_{pi} is analogous to α_r, described in the previous section, for the case where the nonlinear head-dependent (Cauchy-type) boundary represents a discharge-only function. Values for C_{pi} relate the head difference to the volumetric discharge rate and represent the hydraulic characteristics of the spring or drain opening. Usually, the values of C_{pi} are adjusted during calibration until the computed discharge rates match the observed values.

Inputs for nonlinear point sinks consist of the node number where the boundary is located, the reference elevation, z_p, and the value of the discharge coefficient. The number of each boundary node, i, is represented by program variable KP(I), and the reference altitude, z_{pi}, is given by HZP(I). The discharge coefficient, C_{pi}, is represented by program variable GCP. In addition to these inputs, the number of point sinks is input as program variable NPNB and is used to allocate storage for terms associated with this boundary condition. Details about these data inputs are given in the section "Input Instructions."

Steady Vertical Leakage and Evapotranspiration

A nonlinear form of the steady vertical leakage term $R(H - h)$ in equation (1) can be used to limit the head-dependent flux to a maximum recharge or discharge rate over an area or subarea of the aquifer. Some examples of nonlinear steady vertical leakage are: (1) leakage across a confining bed overlying an aquifer that converts between confined and unconfined conditions, (2) leakage through surface-water sediments, such as through a riverbed, when the feature is represented by elements instead of by element sides, and (3) evapotranspiration (fig. 29). For leakage across a confining bed, the head-dependent flux is limited to a maximum inflow (recharge) rate when the aquifer head h is below the altitude of the bottom of the overlying confining bed, z_t (fig. 29*A*). The head-dependent flux across sediments beneath a wide river or lake is limited to a maximum inflow rate when the aquifer head is below the altitude of the bottom of the sediments (fig. 29*B*).

The nonlinear head-dependent flux representing evapotranspiration is a discharge-only type, and is analogous to point sinks or drains, discussed in the previous sections. The evapotranspiration rate, S_e, is

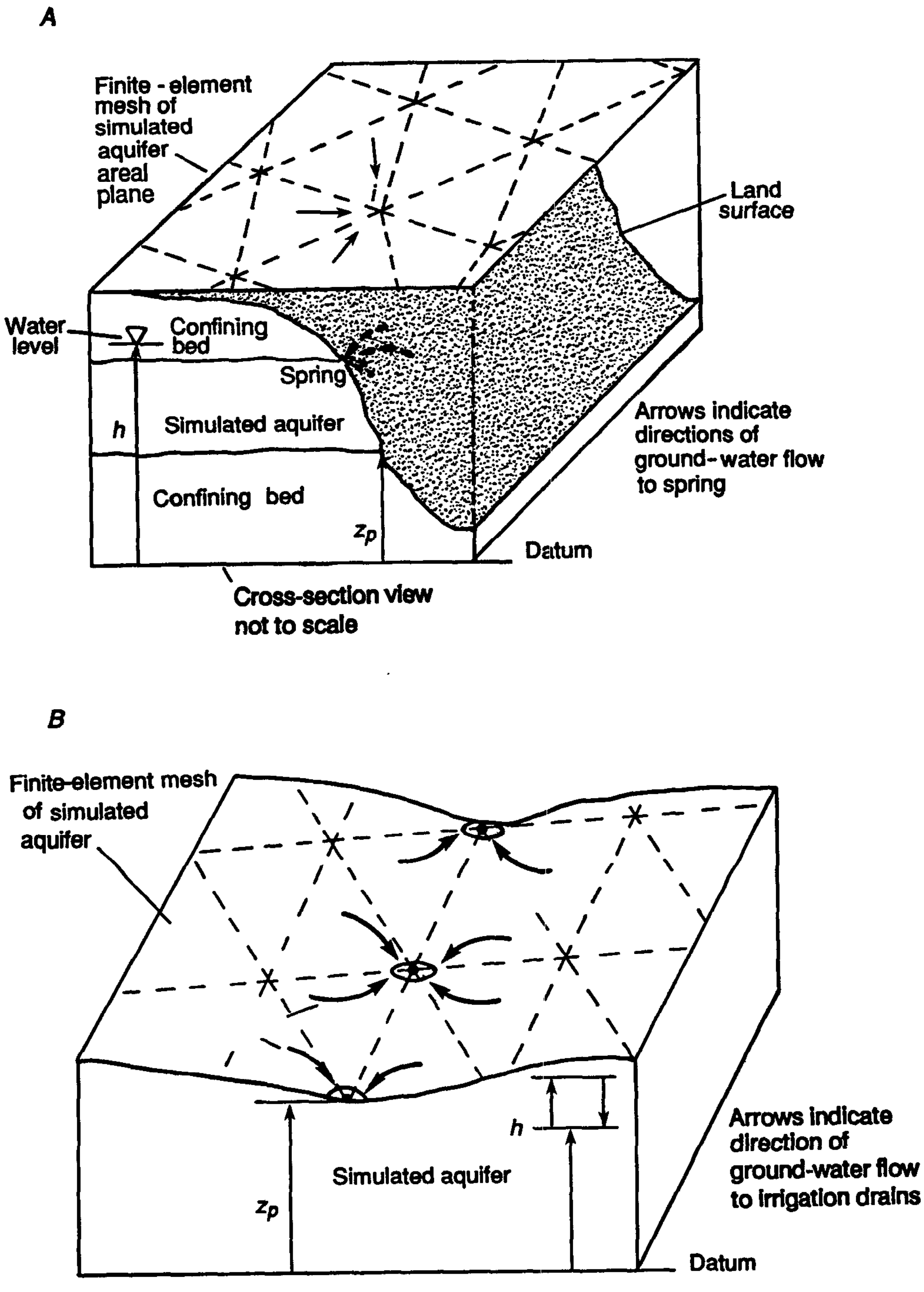


Figure 28.—Nonlinear head-dependent point sinks simulating (*A*) spring at node *i* and (*B*) irrigation drains.

assumed to vary linearly when the aquifer head, h, is situated between land surface and the altitude, z_e, below which the rate is zero. The depth below land surface where S_e is zero is called the extinction depth, d_e. These terms are depicted in figure 29*C*. Evapotranspiration (discharge) occurs at a maximum rate, S_e(max), when the aquifer head equals or exceeds land surface, and is zero when the aquifer head is below z_e,

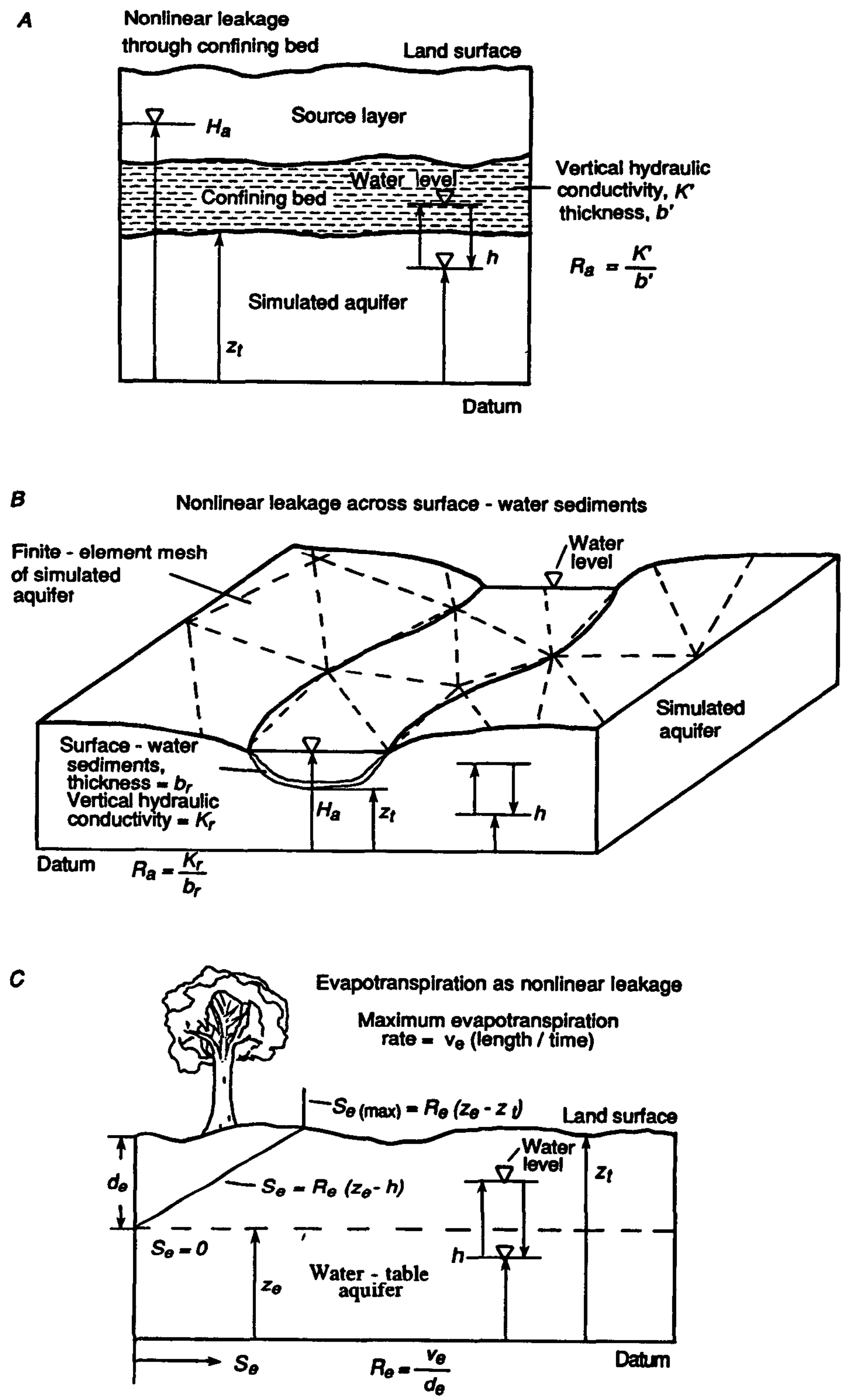


Figure 29.—Nonlinear steady vertical leakage simulating (*A*) flow through confining bed; (*B*) flow across riverbed sediments; and (*C*) evapotranspiration.

which may be the bottom of the root zone. The user may modify the equation-forming subroutine for nonlinear steady vertical leakage if other than linear variation of the evapotranspiration rate with depth below land surface is to be represented. Programming details of the nonlinear head-dependent fluxes are described in Torak (1993).

The expressions for nonlinear steady vertical leakage are given by equation (117) in Cooley (1992), for cases where the maximum inflow rate is limited (examples 1 and 2, above), and by equation (128) in Cooley (1992), for cases where the flux is a discharge-only type (example 3). The terms R_a and R_e, which relate the head differences to the flow rates in these equations, are analogous to the hydraulic conductance R in equation (1) for steady vertical leakage. R_a is computed as the vertical hydraulic conductance of the confining bed or the surface-water sediments; that is, vertical hydraulic conductivity divided by thickness (fig. 29*A*,*B*). R_e is computed as the maximum unit discharge rate, v_e, such as −30 inches/year, divided by the depth interval, d_e (fig. 29*C*). The negative sign is used in the computation of R_e and in the value that is input to MODFE in order to distinguish between the two types of nonlinear steady vertical leakage in the program.

The definition of heads or altitudes that control the nonlinear steady vertical leakage varies depending on the application of the boundary condition. The source-layer head or surface-water level is represented as H_a (fig. 29*A*,*B*). The altitude of the base of the operating zone for the discharge-only function is given as z_e (fig. 29*C*). The altitude of the base of the overlying confining layer or of the surface-water sediments and land surface is defined as z_t.

To simulate nonlinear steady vertical leakage, the area containing the flux is discretized by elements and identified by a unique boundary-condition zone. Details about creating boundary-condition zones are given in the section "Hydraulic Property and Boundary Condition Zones." Elements within a zone can simulate either the condition in which the flux is limited to a maximum rate (examples 1 and 2) or the discharge-only condition (example 3).

Inputs for nonlinear steady vertical leakage consist of the conductance terms R_a or R_e and the controlling heads or altitudes H_a, z_e, and z_t. The conductance terms are represented in MODFE as the program variable VNCF (with negative values for R_e) and are input by boundary-condition zone. The source-layer head or surface-water level H_a and the altitude of the extinction depth z_e is represented by program variable HS, and is input by node. Because nonlinear steady vertical leakage is formulated by using the water-table versions of MODFE, the input of program variable TOP for water-table simulations is used for the altitude of land surface or base of the overlying confining bed, z_e. Thus, nodal values for z_e are not input specifically for this boundary condition. A description of the inputs for nonlinear steady vertical leakage is given in the section "Input Instructions."

Values of the controlling head HS are permitted to change during the simulation to represent a time-varying boundary condition. This feature is useful for varying, for example, surface-water levels or the extinction depth of evapotranspiration to account for seasonal fluctuations in water levels or changes in the ground-water demand by vegetation, respectively. Details of changing HS with time are given in the section "Changing Stresses and Boundary Conditions with Time."

Steady-State Flow

MODFE can be used to solve the two-dimensional ground-water-flow equation (1) for hydraulic heads under steady-state conditions. As discussed in Cooley (1992), steady-state conditions exist, theoretically, when the time derivative of hydraulic head in equation (1) is zero. This causes the storage term $S\partial h/\partial t$ to drop out of the equation and ground-water flow to be independent of aquifer storage and time. The steady-state solution also is independent of the initial conditions of hydraulic head, but requires at least one specified-head boundary (node) or a head-dependent (Cauchy-type) boundary with the nonzero α coefficient to obtain a unique solution.

Because initial estimates of hydraulic head may differ greatly from the actual solution, nonlinear-flow conditions can be simulated prior to achieving a steady-state solution. Examples of nonlinear steady-state-flow problems are water-table (unconfined) conditions, drying or resaturation of parts of the aquifer region, and nonlinear head-dependent flux. These types of aquifer problems are solved by using nonlinear steady-state versions of MODFE, which differ in program structure from the nonlinear, nonsteady-state versions. Linear-flow problems, such as confined conditions with linear boundary conditions are solved by using the linear versions of MODFE, which can solve steady- and nonsteady-state problems. Linear and nonlinear versions of MODFE are listed in Tables 4–6 in the section "Input Instructions" according to their simulation capabilities. Program structures for linear and nonlinear steady-state versions of MODFE are given in the section "Program Structures and Lists of Main Programs" in Torak (1993). Inputs and other information about linear and nonlinear steady-state conditions are discussed in the following sections.

Linear Conditions

Inputs to the linear versions of MODFE determine whether steady- or nonsteady-state conditions are formulated within the program. Steady-state computations are initiated by entering a value of one (1) for the indicator variable ISTD. All values for storage coefficient or specific yield (program variables STR and SY, respectively) are input as zero or left blank. A single time step having a value of one (1.) is required for steady-state simulations. Although steady-state flow is independent of time, a time-step size of one ensures that division by zero will not take place in the equation-forming subroutines, and that values computed for the water-balance summary represent volumes for a unit-time interval. Details of these and other data inputs are given in the section "Input Instructions."

Nonlinear Conditions

Nonlinear conditions of steady-state ground-water flow are represented in MODFE by an iterative-solution method that contains data inputs and equation formulations that differ from the nonlinear, nonsteady-state versions. The iterative method contains water-table iterations during which aquifer thickness and terms associated with nonlinear boundary conditions are updated by using head changes from the previous (or initial) iteration level. Details of the iterative scheme and of these updates are given in the sections "Nonlinear Case" (Cooley, 1992) and "Nonlinear Conditions" (Torak, 1993).

The nonlinear steady-state versions of MODFE require the following inputs for defining the iterative-solution method: values for the maximum number of water-table iterations, maximum allowable displacement, and closure tolerance for steady state. The maximum number of water-table iterations is represented by the program variable NITSW. The number of iterations needed to reach an acceptable nonlinear solution varies for each simulation, and is dependent on the number of nonlinear conditions that exist in the aquifer problem, the maximum allowable displacement, and the closure tolerance. Most nonlinear steady-state problems can be solved with fewer than 50 water table iterations.

The maximum allowable displacement, or head change, is represented by the program variable DSMX. Head changes greater than DSMX that are computed during an iteration level are damped (decreased) before they are used to update aquifer thickness and to evaluate nonlinear-boundary conditions. For highly nonlinear problems; that is, simulations involving many nodes that require evaluation of nonlinear conditions, values of DSMX should be selected smaller than about half of the expected head change from the initial to the final conditions. Specification of DSMX in this manner allows updates to nonlinear terms to be made gradually over several water-table iterations, instead of making large changes to nonlinear terms over a few iterations. This latter condition may cause inappropriate updates to the nonlinear terms that can lead to excessive iteration, nonclosure, or closure to an incorrect solution.

The closure tolerance for steady-state is represented in MODFE by the program variable TOLSW, and can be selected so that an acceptable solution to the nonlinear problem is obtained with a minimum number of iterations. Values for TOLSW may be selected initially as an order of magnitude smaller than the accuracy of the observed water levels, and then adjusted to give the desired level of acceptability of flow rates and flow imbalance in the water-balance summary. Usually, the nonlinear solution is accepted when the value for the flow imbalance that is output in the water-balance summary is about five to six orders of magnitude smaller than the largest flow rate. A discussion of mass-balance and error terms is given in the section "Water-Balance Summary and Flow Imbalance" in Torak (1993).

Values for TOLSW may be selected differently than described above if the MICCG method of solution is used. Although the closure tolerance for steady state is independent of the convergence criterion ϵ for the MICCG method (see equations (285) and (289) in Cooley, 1992), the relative values for each criterion may be adjusted to achieve faster convergence than if only TOLSW were adjusted. Selection of values for both closure criteria is discussed in the section "Stopping Criteria," in Cooley (1992).

Other data inputs to the nonlinear steady-state versions of MODFE are similar to the inputs required for linear steady-state versions. These are: a value (1) for the indicator variable ISTD, discussed in the previous section, and nodal values of aquifer thickness (THK) and altitude of the top of the water-table aquifer or the bottom of an overlying confining bed (TOP). Values for the storage coefficient (STR) and specific yield (SY) should be set to zero or left blank. Additional information on these data inputs are given in the section "Input Instructions."

Selecting Stress Periods and Time-Step Sizes

The selection of stress periods and time-step sizes is dependent entirely on the distribution of aquifer stresses over time. A stress period usually defines a time interval over which aquifer stresses and boundary conditions are constant. Stress periods are subdivided into time steps, and solution to the finite-element equations is provided by MODFE for average conditions that exist during the time step. The ability to change stresses and boundary conditions in MODFE for any time step may appear to make stress-periods obsolete. However, changes to particular stresses, such as well-pumping rates, or to boundary conditions that create stress, such as surface-water levels, may occur with a degree of regularity or irregularity, or may define an historic event that requires identification as a stress period, rather than as a change in stress on a time-step basis. For example, ground-water withdrawals may have increased suddenly within a short period of time due to increased demands by population, agriculture, or industry. Thus, one stress period can be established to represent the time prior to the increase in pumping rates, and another stress period can be used to represent the time after the increase was invoked. However, within these stress periods, surface water levels or boundary fluxes also may vary, or a few wells could either cease or begin pumping. It may be determined that these changes either impose minor effects on the aquifer or occur frequently so that they can be represented by time-step variations in stresses or boundary conditions. Thus, for this example, stress periods are established to coincide with major changes in stress on the aquifer.

The use of many stress periods, each containing the same number and size of time steps, may have computational advantages in MODFE over using one stress period having a long series of time steps. If the number and size of the time steps are identical for successive stress periods, then the time-step information from the first stress period that contains identical time-step information as the previous stress period can be used on subsequent stress periods without inputting new time-step data. Also, computer storage is decreased because only the time-step sizes for the first stress period in the succession of identical stress periods are required to be stored. This is accomplished in MODFE by evaluating the indicator variable NTMP at the beginning of the inputs for the stress period (see section "Input Instructions"). A value of zero (0) for NTMP will permit the time-step number and sizes from the current stress period to be used for the following stress period. The savings in computer storage can be large depending on the number of stress periods that can be formed in this manner.

The selection of time-step sizes may have a more theoretical basis than the establishment of stress periods. Of equal importance as spatial discretization of finite-element equations is the discretization of the finite-element equations in time. Although problem dependent, some guidelines can be applied to selecting time step sizes that ensure adequate discretization of the time derivative in equation (1).

Important considerations when selecting time-step sizes are the magnitudes of the initial stresses (or boundary conditions), changes to stresses (or boundary conditions), and the anticipated effects on the aquifer. Large stresses or changes in stress relative to the aquifer's ability to respond may require many small time steps during the initial stages in which the stress is applied in order to adequately represent nonsteady-state flow. Aquifer response may be such that nonsteady-state gradients may be established only a short distance from the stress, and that the aquifer will exhibit nonsteady-state conditions for a long period of time. Small time steps are needed to accurately approximate head changes during nonsteady-state conditions in a manner analogous to the increased discretization by small-area elements that is needed in the vicinity of a pumped well. Time-step sizes can increase from the small sizes that are needed initially to adequately simulate aquifer response to a new or changed stress, to gradually larger time steps as the aquifer response diminishes and steady-state conditions are reached. Accurate results have been obtained by gradually increasing time steps by a factor of 1.1 to 1.5 of the previous value.

The effects of time-step sizes on the solution of hydraulic head and on volumetric flow rates for mass-balance terms can be determined by performing a sensitivity test. Time-step sizes are decreased (or increased) from their initial or previous values and the aquifer problem is simulated again. Values for the volumetric rate of accumulation and total volume of water in aquifer storage, which are computed in the water-balance summary, are noted for each simulation. Simulations are performed until there is no significant change in these values when compared with similar values from the previous simulation. Appropriate time-step sizes for subsequent simulations are determined as the largest time steps that allow maximum values for the volumetric rate and total volume of water in aquifer storage; additional discretization in time (smaller time steps) yield no significant increase in the volumetric rate and total volume of water in aquifer storage.

Proper selection of time-step sizes will provide accurate representation of time-variant terms in nonlinear-flow problems. Solutions to flow problems involving water-table conditions, conversion between confined and unconfined conditions, drying and resaturating parts of the aquifer, and nonlinear head-dependent flux are particularly sensitive to the time-step size. Time-step sizes for nonlinear-flow problems can be smaller than those that are used for linear problems because of the time variation of terms used to form the finite-element equations. Adequate time-step sizes for nonlinear problems can be determined by conducting the sensitivity test described above and by evaluating the flow imbalance which is printed in the water-balance summary. In addition to the criterion described above, suitable time-step sizes are the largest values that allow a flow imbalance to be computed to within the accuracy of computer; that is, about six orders of magnitude smaller than the largest volumetric flow rate in the water-balance summary. Because nonlinear conditions may exist during the entire simulation period, increases to time-step sizes may not be appropriate or may have to be kept small.

Changing Stresses and Boundary Conditions with Time

Stresses, such as point sources and sinks, specified fluxes, and areally distributed fluxes; controlling heads to boundary conditions, such as specified heads; and boundary and external heads to linear and nonlinear head-dependent fluxes, are allowed to vary during a simulation as functions of time. Changes to these terms can be made at any time step of any stress period as determined by the aquifer problem. Depending on the stresses or boundary conditions to be changed, MODFE is structured to contain up to four additional subroutines, and the corresponding Fortran call statements, to perform the related computations and data inputs. In addition, most of the stresses and boundary conditions that are permitted to change with time require two time steps to implement the change (Table 1). Details of these computations are given in the section "Changing Stresses and Boundary Conditions with Time" in Torak (1993). Program structures are given in the section "Program Structures and Lists of Main Programs" in Torak (1993).

Time variance of stresses and boundary conditions are represented in finite-element matrix equations (254) and (257) in Cooley (1992) by the vector B, which is defined by equations (60)–(62) in Cooley (1992). This formulation requires that the average value of the stress or boundary condition be used when forming the matrix equation. Consequently, average values for the changed stress or boundary condition must be computed by the user and input to MODFE for the time step that the change occurs. On the time step following the change, the user must input the new value for the changed stress or boundary condition, as the average value for the new time step is now the changed value. Thus, changes to stresses and boundary conditions take two time steps to invoke.

Changes to stresses and boundary conditions by using the two-time-step procedure described above are demonstrated by the following example. Consider changing the external head, H_B, on a head-dependent (Cauchy-type) boundary from its current value of 100 feet to 109 feet on the following time step of a simulation. The user computes the average value of H_B according to the expression for the average B vector, equation (62) in Cooley (1992), as 106 feet, or, $(1/3) \times 100$ feet + $(2/3) \times 109$ feet. This value (106 feet) is input to MODFE on the first time step in which the change is take effect. On the following time step, the user computes the average value of H_B as 109 feet, $(1/3) \times 109$ feet + $(2/3) \times 109$ feet, and inputs the new value of H_B (109 feet) to MODFE. Subsequent time steps use the value of 109 feet for H_B until another change takes place. If changes to H_B are made on successive time steps, then average values for H_B are computed by the user according to equation (62) in Cooley (1992) and are input on every time step in which a change is to take effect. The last 'new' value for H_B is input on the time step following the last change.

An exception to using two time steps for changing boundary conditions as described above applies to changing values of head at specified-head boundaries and source-layer heads that are used to simulate transient leakage (see Table 1). For these two cases, the average head is computed automatically within MODFE according to equation (62) in Cooley (1992). Thus, the user inputs only the new values for these heads; changes to these boundary conditions are made in MODFE by using one time step.

Changes to point sources and sinks and areally distributed fluxes require the old value to be input along with either the average or the new value when a change is made (Table 1). The old values are needed to adjust the appropriate mass-balance terms, as the known fluxes are not represented by individual program variables at each node.

Each stress or boundary condition that is allowed to change during the simulation has an indicator variable assigned to it. The values for the indicator variables define the time step on which the change is to occur. On the appropriate time step, inputs corresponding to changes in stresses or boundary conditions are read by MODFE. In addition to these inputs, a new value for the indicator variable is input to allow changes on subsequent time steps. If two time steps are required to change a stress or boundary condition, then the indicator variables contain the appropriate time-step numbers that would permit changes to be made on successive time steps. Although it is possible to input initial conditions of stresses or boundary conditions by indicating a change on the first time step of the first stress period (indicator variables set to 1 followed by the appropriate inputs), initial values are input more

Table 1.—Program variables and subroutines used to change stresses and boundary conditions with time

Stress or boundary condition	Number of time steps to implement change	Indicator variable	Old value	New value	Subroutine
Source-bed head for transient leakage	1	NCBCH	--	HRJ	CBCHG
Point sources/sinks	2	NWCH	QOLD	QNEW	COCHG
Areally distributed recharge/discharge	2	NQCH	QOLD	QNEW	COCHG
Source-bed head for steady leakage	2	NHRCH	--	HR(J)	COCHG
Specified-flux boundary	2	NBQCH	--	QNEW	COCHG
Head-dependent (Cauchy-type) boundary	2	NBQCH	-- --	HK(J) HL(J)	COCHG
Specified-head boundary	1	NHCH	--	HB	COCHG
Nonlinear head-dependent (Cauchy-type) boundary	2	NGNCH	-- --	HRK(J) HRL(J)	GNCHG
Controlling head for nonlinear steady leakage	2	NVNCH	--	HS(J)	VNCHG

easily by using the input mechanisms designed for initial conditions (see section "Input Instructions" for descriptions of indicator variables and other inputs) than the subroutines and inputs designed to change these values.

Inputs consisting of the changed values for stresses or controlling heads are made on each time step according to the sequence given in the input instructions. Entries are omitted from the sequence of inputs for a particular time step if the related stress or controlling head is not to be changed. Conversely, multiple inputs of changes to one type of stress or boundary condition may precede entries for other types, if a particular change is made more frequently than others. Details of these inputs are given in the sections "Input Instructions" and "Examples of Model Input."

Data Preparation

All data that are required for simulating a given aquifer problem are prepared for input to MODFE according to the instructions given in the section "Input Instructions." However, the input of hydraulic properties, boundary conditions, and the finite-element mesh to MODFE can be simplified. Hydraulic properties and boundary conditions that are input by element and element side can be grouped by common values into hydraulic-property and boundary-condition zones. Another simplification involves combining the input of node numbers that define elements (element incidences) so that one set of incidences define two triangular elements. The following sections describe how to combine element incidences, establish hydraulic-property and boundary-condition zones, and input values to MODFE.

Combined-Element Incidences

The node numbers that define an element, termed element incidences, can be combined for two contiguous elements for input to MODFE. Combining element incidences simplifies input, decreases computer-storage requirements, and utilizes the efficient programming style of MODFE most effectively. To combine element incidences, the four node numbers that define an element pair (two contiguous elements) are written in counterclockwise order (fig. 30*A*). The element pair is divided into two triangular elements (fig. 30*B*) along the element side defined by the first and third element incidences (nodes 35 and 42 in fig. 30*A*).

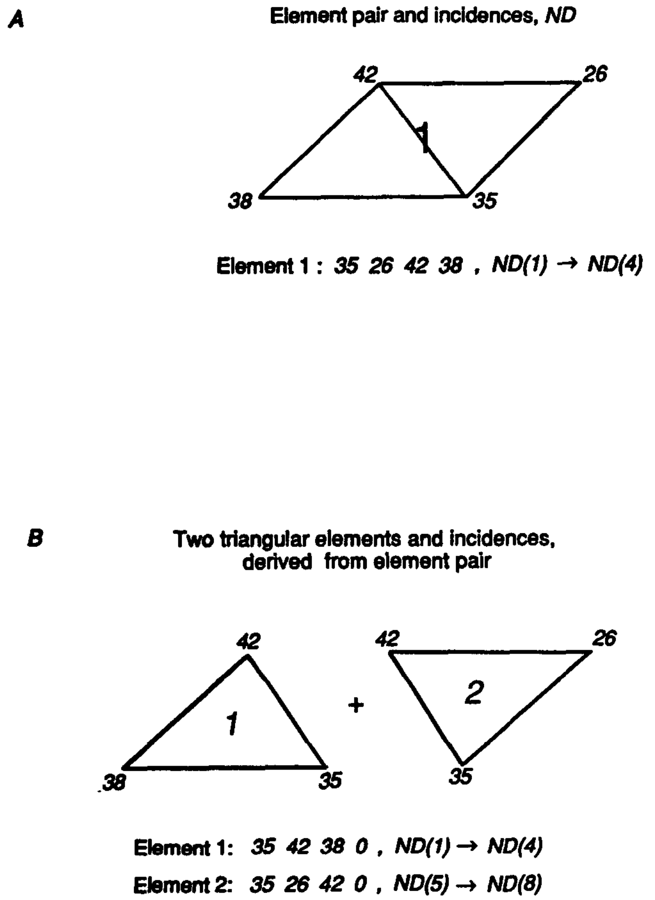


Figure 30.—Input of element incidences to MODular Finite-Element model (MODFE) for (*A*) two contiguous elements having combined element-incidences and (*B*) two elements having separate element incidences.

Element incidences are represented in MODFE by the program variable ND. MODFE is designed to use combined-element incidences; thus, all elements or element pairs require four values of incidences. For a triangular element (fig. 30*B*), the fourth entry of the incidences is zero. Note that by combining element incidences, four nonzero values, ND(1) through ND(4) (fig. 30*A*), can define the element pair, whereas eight values, ND(1) through ND(8) (fig. 30*B*), are required to define individually the two triangular elements. Both methods shown in figure 30 for representing two triangular elements with incidences are equivalent, and both methods can be used simultaneously within the same finite-element mesh.

Savings in computer storage and computational efficiency is realized by using combined-element incidences. The amount of storage needed for the program variable ND is decreased by combining the input of element incidences. The element pair is counted as one element when determining the number of elements in the mesh. Thus, computer storage is decreased for program variables that are dimensioned in terms of the number of elements. Computations for element areas and nodal terms in the finite-element equations are performed more efficiently when element incidences are combined than if incidences were input separately.

Because combined-element incidences cause the element pair to be counted as one element, both triangles of the element pair contain the same values for hydraulic properties and for the rotation angle of anisotropy. Consequently, both triangles of the element pair are located within the same hydraulic-property zone. Details of establishing hydraulic-property zones are given in the following section.

Hydraulic-Property and Boundary-Condition Zones

Inputs for hydraulic properties and boundary conditions that are made to MODFE by elements or by element sides can be simplified greatly by grouping elements or element sides into zones. A zone is a group of elements or element sides that requires the same values to define hydraulic properties or boundary conditions. A zone can contain one element (or side) or as many as all elements (or sides) in the mesh. Different types of zones are used in MODFE to simplify data input. Descriptions of zones that are created by grouping elements are given first. These are followed by descriptions of zones that are created by grouping element sides. Examples of establishing zones for an aquifer problem are given after these descriptions.

One type of hydraulic-property zone contains elements that have identical values for the following information:

- aquifer transmissivity or hydraulic conductivity in the x (or horizontal) and y (or vertical) directions,
- rotation angle for transforming x-y coordinates to local coordinates for anisotropic flow,
- vertical hydraulic conductance of a confining bed (linear case),
- aquifer storage coefficient (or specific storage for cross-section or axisymmetric flow), and
- areally distributed stresses.

Another type of element zone groups elements according to the value of one hydraulic property. Grouping of this type may be used for inputting the following hydraulic properties:

- specific yield for water-table simulations,
- specific storage and vertical hydraulic conductivity for transient leakage, and

- coefficient (R_a or R_e) for nonlinear steady vertical leakage or evapotranspiration.

Boundary conditions may be input by zone either to group element sides that contain the same value for terms defining the hydrologic condition or to identify a group of element sides that require distinct values for boundary-condition identification. Zones may be created for the following boundary conditions:

- specified flux (linear case),
- linear, head-dependent (Cauchy-type) flux, and
- nonlinear, head-dependent (Cauchy-type) flux.

The establishment of hydraulic-property and boundary-condition zones for data inputs to MODFE is demonstrated with the following example. The area of interest contains a water-table aquifer that is dissected by two rivers (fig. 31*A*). Aquifer heads are affected by steady vertical leakage (no storage effects) through an underlying confining bed. Alluvium in the valley of one of the rivers creates confined- and semiconfined-aquifer conditions with the potential for conversion between confined and unconfined conditions. There is head-dependent (regional) inflow of ground water from the northeast and outflow to the southwest.

The pertinent hydraulic properties and boundary conditions to be considered for zoning are the aquifer hydraulic conductivity, vertical hydraulic conductances of the underlying confining bed and of the alluvium, and the α coefficients for the head-dependent (Cauchy-type) boundaries, which represent the rivers and regional flow. Distributions of these hydraulic properties and boundary conditions are shown in figures 31 and 32.

The finite-element mesh of the area of interest is shown in figure 33A. Note the finer discretization by elements that are located in areas covered by alluvium and near the inflow boundary. It is anticipated that the aquifer head will be more time variant in these areas than in other areas because of conversion between confined and unconfined conditions and because of varying boundary inflow.

The zoning process begins by superposing the finite-element mesh on the distributions of hydraulic properties and boundary conditions. Element sides that approximate the boundaries of each distribution are identified (figs. 33 and 34). Nodes are moved from their original positions in the mesh so that locations of hydraulic-property zones and boundary conditions are defined by element sides.

Grouping Elements into Zones

Zones containing elements that have identical values for the hydraulic properties listed at the beginning of the section "Hydraulic-Property and Boundary-Condition Zones" are created first by defining boundaries for the distributions within each property (by using element sides) and then by combining the boundaries of different properties. For the example aquifer problem, boundaries for the distributions of aquifer hydraulic conductivity and vertical hydraulic conductance of the underlying confining bed (figs. 33*B* and 34*A*) are combined to form hydraulic-property zones for input to MODFE. Also, for this example, it is assumed that the other hydraulic properties in the list, which could be used to create other zone boundaries, are either constant over the aquifer area or have a value of zero. Thus, the intersection of boundaries for aquifer hydraulic conductivity and vertical hydraulic conductance create seven hydraulic-property zones (fig. 35*A*); all elements within each zone have the same values of vertical hydraulic conductance and hydraulic conductivity.

Inputs for each hydraulic-property zone consist of values that define the zone number, number of elements in the zone, and hydraulic properties, followed by the element numbers and element incidences (node numbers that define the element). The zone number and the number of elements contained in a zone are represented in MODFE by the program variables KZ and NO, respectively. The hydraulic properties that are listed in the previous section for hydraulic-property zones are represented by the variables XTR, YTR, ANG, VLC, STR, and QD, respectively. The element number is represented by the program variable IEL, and element incidences are represented by the program variable ND. Four values of incidences are used for each element instead of three because MODFE permits the incidences of two adjacent elements to be combined, or paired, for input (see section "Combined-Element Incidences"). An example of the inputs for hydraulic-property zone 1 containing 120 elements or element pairs is given in figure 35*B*.

Elements are identified within hydraulic-property zones and sequenced in the finite-element mesh according to the order in which the incidences are input. For zone 1, the incidences for all 120 elements (or element pairs) are input consecutively following the input of hydraulic properties for the zone. Thus, 120 sets of 4-node incidences, or 480 contiguous storage locations for ND, define nodes that correspond to elements or element pairs within zone 1 and also define the first 120 elements or element pairs in the mesh. The first four incidences are associated with the first element or element pair in the mesh, and the last four incidences (for zone 1) are associated with the 120th element or element pair.

Element numbering within a zone can be arbitrary as element numbers are not used in MODFE. However, the element numbers that are printed by MODFE correspond to the order in which the inci-

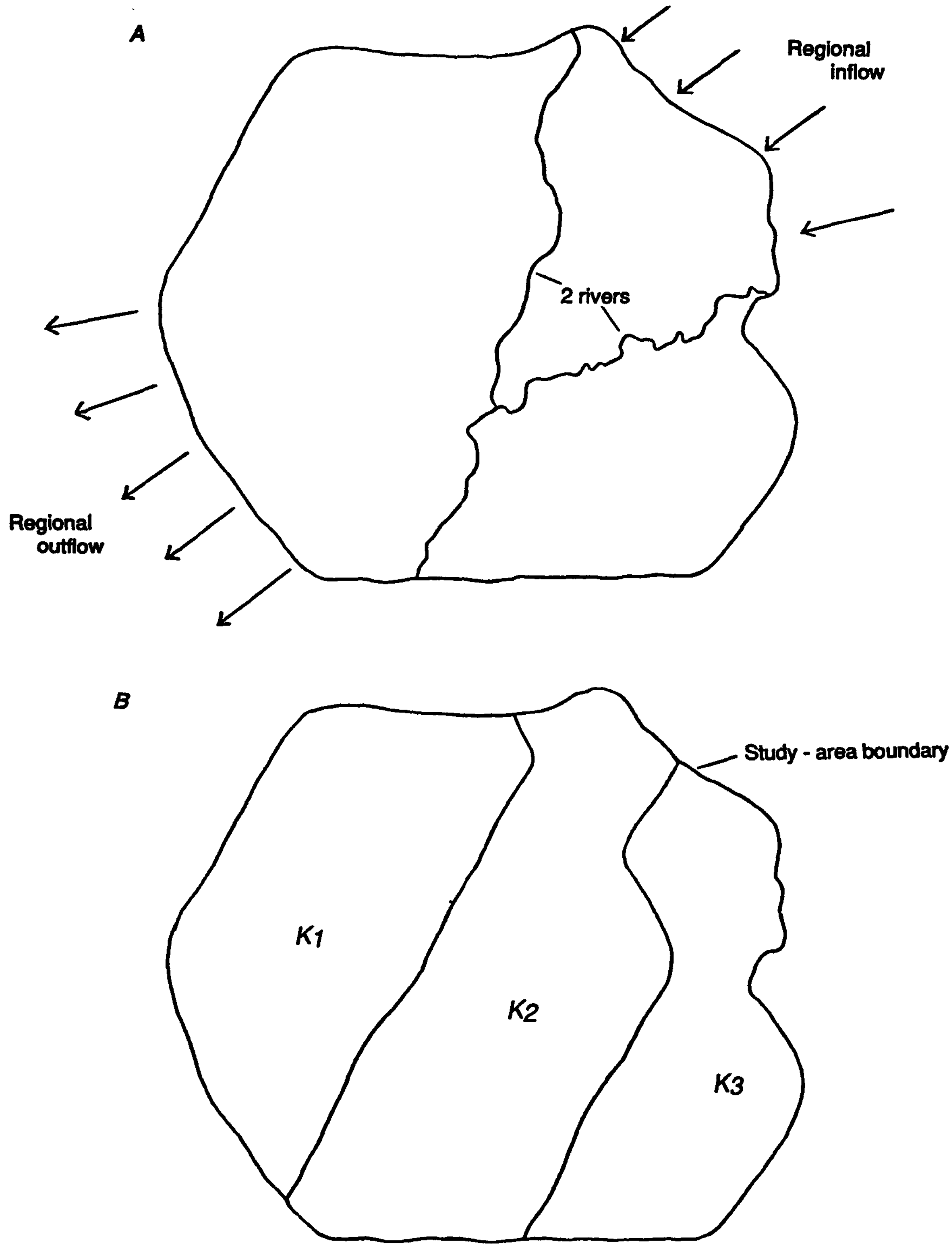


Figure 31.—(*A*) Areal representation of water-table aquifer dissected by two rivers and (*B*) hydraulic conductivity zones, *K1*, *K2*, and *K3*, for water-table aquifer.

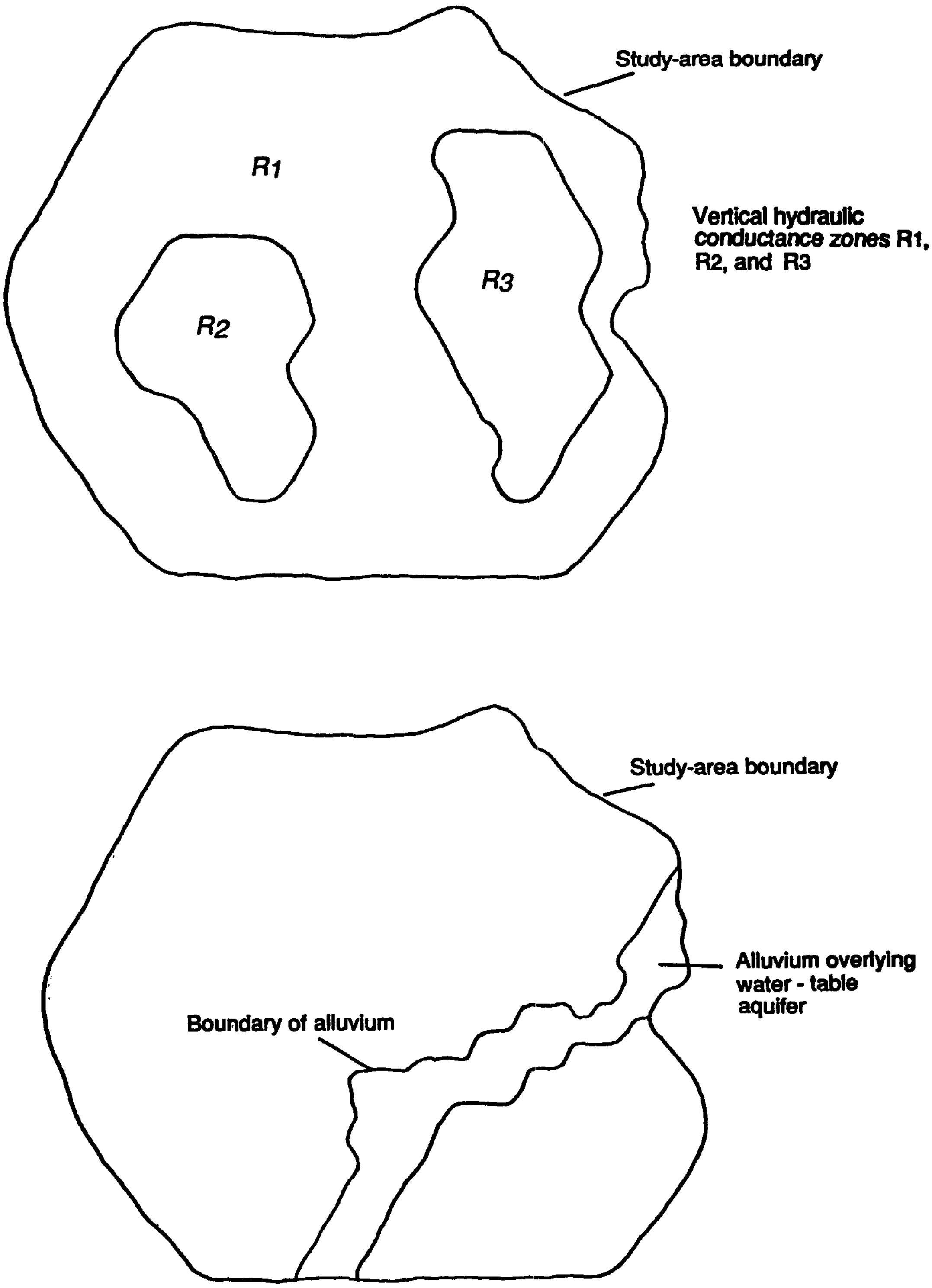
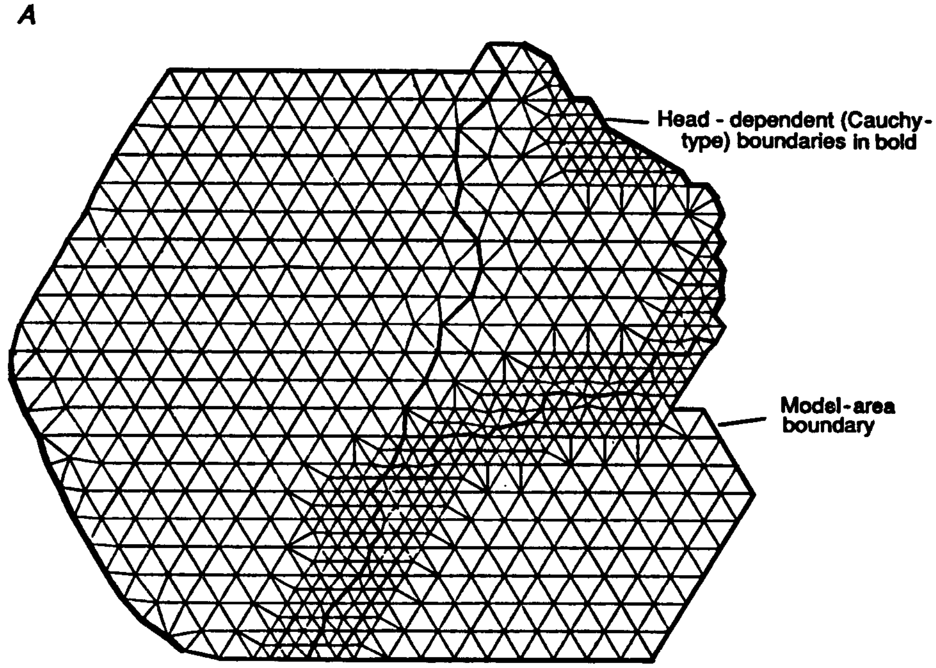


Figure 32.—Areal distribution of vertical hydraulic conductance for (*A*) confining bed underlying water-table aquifer; and (*B*) alluvium overlying water-table aquifer.



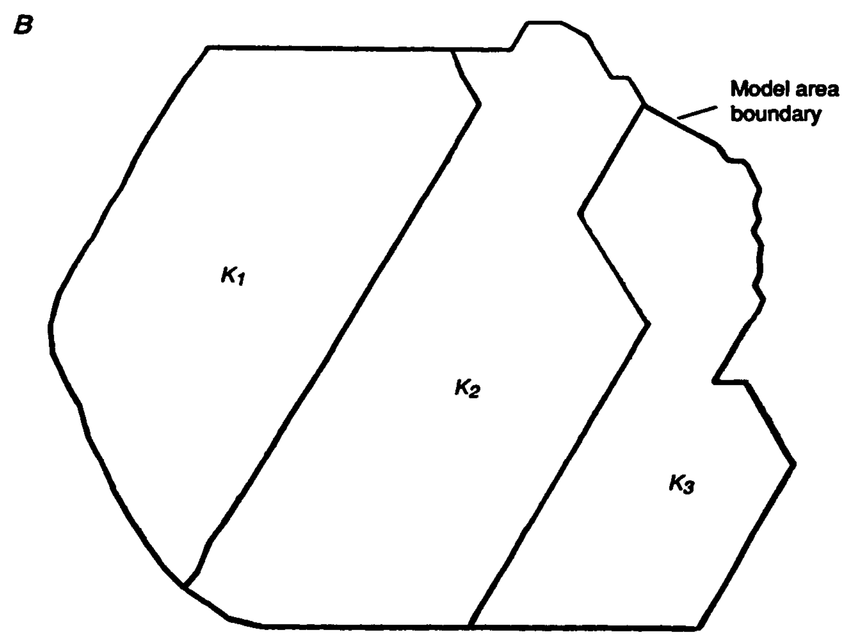


Figure 33.—(*A*) Finite-element mesh for example aquifer problem and boundary conditions and (*B*) hydraulic-conductivity zones, *K1*, *K2*, and *K3*, bounded by element sides.

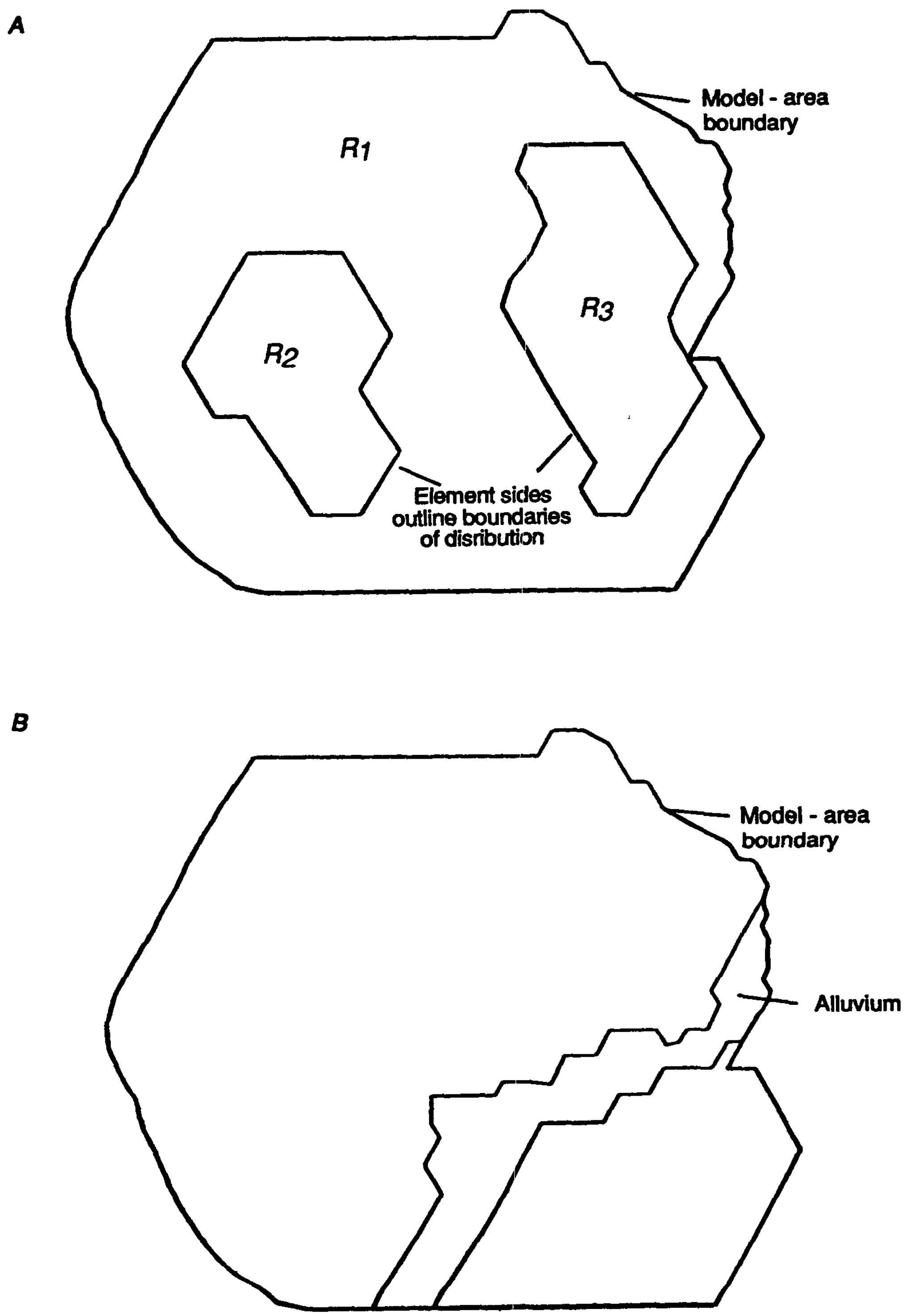
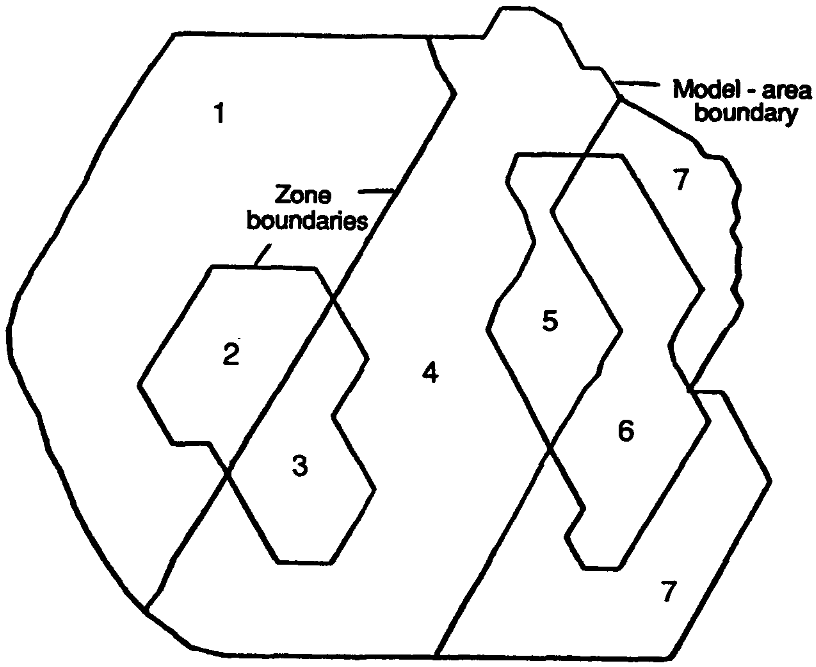


Figure 34.—Element sides used to represent (*A*) vertical-hydraulic conductance zones, *R1*, *R2*, and *R3*, for confining bed underlying aquifer and (*B*) boundary of alluvium overlying water-table aquifer.

A



B

KZ	NO		XTR		YTR	ANG	VLC	STR	QD
1	120		1200.		1200.	0.	1.E-2	1.E-4	0.
1	30	14	31	55					
2	14	4	15	31					
3	4	1	5	0					
4	15	5	8	20					
5	17	18	6	16					
.	.	.	.	.					
.	.	.	.	.					
.	.	.	.	.					
116	18	19	7	0					
117	16	6	4	14					
118	6	7	1	0					
119	2	8	5	1					
120	97	68	100	138					
IEL	ND(I), I=1,480								

Figure 35.—(*A*) Hydraulic-property zones (7) resulting from intersecting boundaries for vertical hydraulic conductance and aquifer hydraulic conductivity and (*B*) example of input and program variables for zone 1.

dences have been input. Because the finite-element equations are formed and solved at nodes, it is the node numbers (defined by the element incidences), not the element numbers, that determine where computations will occur within a zone.

Zones may be input in any order, that is, values for zone 1 do not have to be input first. In this manner, the order of the element incidences within a hydraulic-property zone and the order of the zones can be used to define additional zones for nonlinear steady vertical leakage, transient leakage, and specific yield, discussed in the following section.

Zones for Nonlinear Steady Vertical Leakage, *Transient Leakage, and Specific Yield*

The order in which element incidences are input for hydraulic-property zones can be used to create other zones for nonlinear steady vertical leakage, transient leakage, and specific yield. The example-aquifer problem is used to demonstrate the creation of zones for simulating nonlinear steady vertical leakage between the water-table aquifer and the alluvium. By superposing the distribution of vertical hydraulic conductance of the alluvium (fig. 34*B*) on the hydraulic-property zones (fig. 35*A*), five zones for nonlinear steady vertical leakage are created from the intersecting boundaries (fig. 36*A*). However, reordering the input of element incidences within hydraulic-property zones 3 through 7 results in creating only three zones of nonlinear steady vertical leakage (fig. 36*B*). (Fewer zones decrease the number of changes to be made during calibration, and hence, decrease the potential for making errors when changing zone values.)

One zone for nonlinear steady vertical leakage consists of the elements within hydraulic-property zones 3 and 4 that contain alluvium (fig. 36*A*). The inputs of hydraulic-property data and incidences are arranged so that incidences for elements corresponding to nonlinear steady vertical leakage are placed in contiguous storage locations within the program vector ND. That is, incidences of elements in hydraulic-property zone 3 that contain alluvium are input last in the incidence list for zone 3, and incidences for elements that contain alluvium in zone 4 are input first in the incidence list for zone 4. Thus, a contiguous set of node numbers from hydraulic-property zones 3 and 4 corresponding to the nonlinear vertical leakage zone is stored in ND. If the number of elements in hydraulic-property zones 3 and 4 that contain nonlinear steady vertical leakage are 2 and 70, respectively, then the leakage zone would contain 72 elements (assuming the data for zones 3 and 4 are input consecutively).

Another vertical hydraulic-conductance zone can be created by ordering the incidences of elements within hydraulic-property zones 5 and 6 that contain nonlinear steady vertical leakage. Incidences of elements in hydraulic-property zone 5 that contain nonlinear steady vertical leakage are input last in the incidence list for zone 5. These incidences are followed by those in zone 6 that contain nonlinear steady vertical leakage, thereby creating a contiguous set of storage locations in ND that contain node numbers of elements in the nonlinear-leakage zone. Note that elements in the nonlinear-leakage zone can be separated in the finite-element mesh from other elements in the same zone as long as their incidences are input consecutively. A third vertical hydraulic-conductance zone for nonlinear steady vertical leakage can be created by using elements in hydraulic-property zone 7 that contain the alluvium.

Inputs for nonlinear steady vertical leakage and transient leakage are used to identify contiguous storage locations within ND for establishing zones and for assigning values to hydraulic properties. The inputs consist of values for the beginning element number in the leakage zone, number of elements contained in the zone, and, depending on the physical process that is simulated, vertical hydraulic conductance for nonlinear steady vertical leakage and (or) vertical hydraulic conductance and specific storage of the confining bed for transient leakage. The beginning element number of the zone is input as the program variable NBE, and the number of elements in a zone is input as the program variable NO. The zone number is represented by the program variable L. For nonlinear steady vertical leakage, vertical hydraulic conductance is represented by the program variable VNCF, and for transient leakage, vertical hydraulic conductance and specific storage of the confining bed are represented, respectively, by the program variables VCON and SPST. Descriptions of these inputs are given in the section "Input Instructions."

The value of NBE is used in MODFE to locate the incidences associated with elements in zone L. For instance, the value of NBE-1 gives the number of sets of 4-node incidences (stored in the program vector ND) that precede the incidences associated with zone L. The value of NO is used to determine the number of sets of incidences contained in zone L, which are evaluated by MODFE when terms for either nonlinear steady vertical leakage or transient leakage are formed for the finite-element equations.

The use of NBE and NO in MODFE is demonstrated in the example-aquifer problem described above and in some of the inputs for creating nonlinear

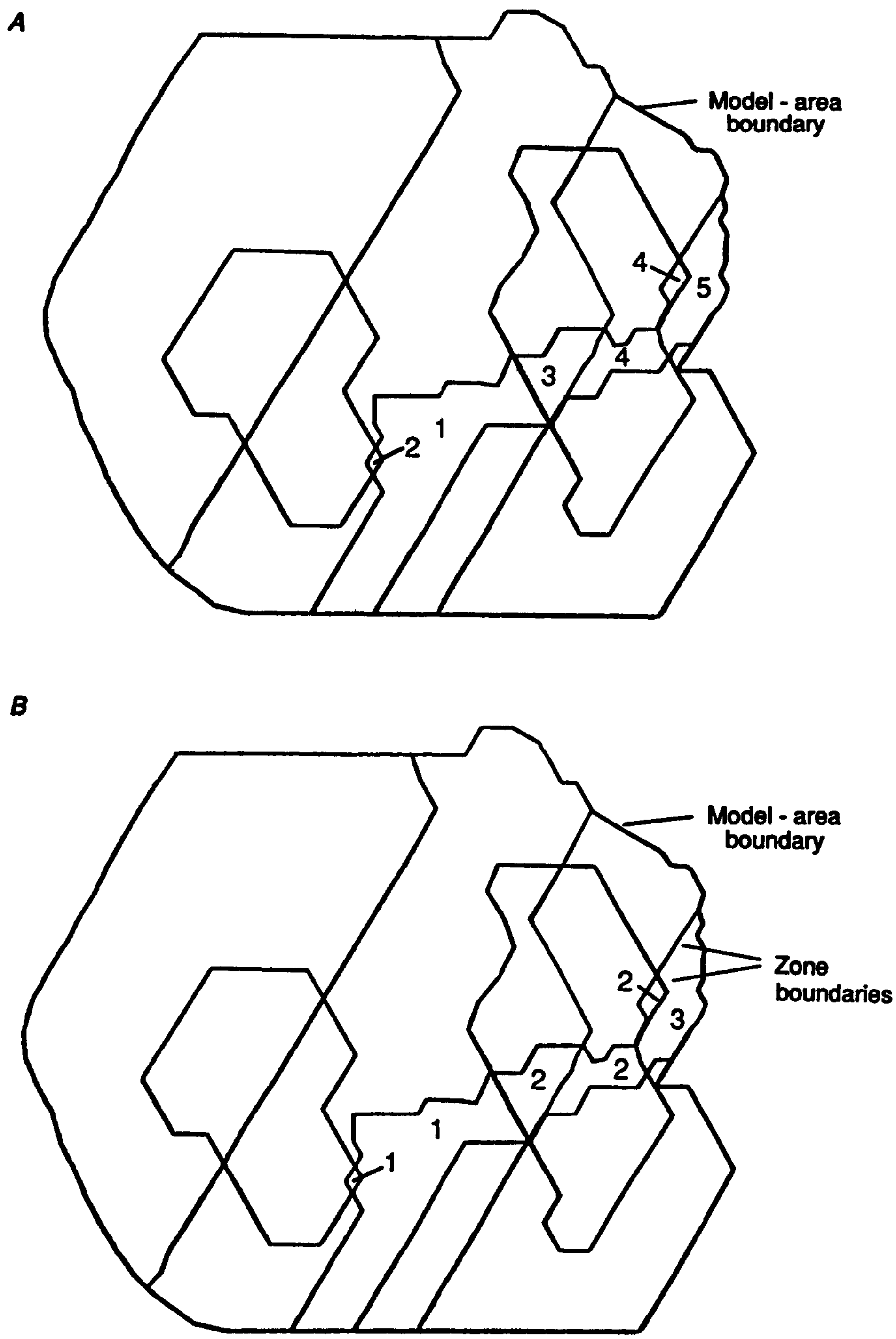


Figure 36.—Areal representation of simulated aquifer showing effect of ordering element incidences on establishing zones for nonlinear steady vertical leakage; (*A*) no ordering; (*B*) ordered incidences.

steady vertical-leakage zones (Table 2). The notation to the right of the incidence lists defines the values for NBE and NO that are used to locate incidences associated with nonlinear steady vertical-leakage zones. Assume that inputs for the hydraulic-property zones were made sequentially (1 through 7), and that element incidences were arranged to create three nonlinear steady vertical leakage zones. Inputs defining the beginning element and the number of elements within each nonlinear steady vertical leakage zone are listed in Table 2A as values for program variables NBE and NO, respectively. Thus, there are 157 elements contained in the three nonlinear steady vertical leakage zones.

From Table 2A, the beginning element number, NBE, for the first nonlinear steady vertical leakage zone is 156, and the number of elements, NO, in the leakage zone is 72. Element incidences for hydraulic-property zones 3 and 4 are arranged so that the last two sets of incidences in zone 3 (for elements 156 and 157) and the first 70 sets of incidences in zone 4 contain the values that correspond to elements in the first zone of nonlinear steady vertical leakage (Table 2B). Similar arrangement of element incidences and specification of NBE and NO define the other two nonlinear steady vertical-leakage zones (Table 2).

Zones of specific yield for water-table simulations are identical to hydraulic-property zones, discussed in the section "Hydraulic-Property and Boundary-Condition Zones." Therefore, only values for the number of elements in a zone, NO, are required to achieve the appropriate correspondence of specific yield to nodes. Values of NBE are not required because specific yield is assigned to nodes within each element in the finite-element mesh according to the order of element incidences that was previously established by the hydraulic-property zones. Thus, hydraulic-property zones and element incidences are arranged in such a manner that a minimum number of zones for specific yield is created and all elements are assigned a value (see section "Input Instructions" for details).

Grouping Element Sides into Zones

The bold element sides in figure 33*A* represent head-dependent (Cauchy-type) boundaries that simulate rivers and boundary flows. Seven boundary-condition zones result from intersecting the bold element sides with the boundaries of the hydraulic-conductivity zones and alluvium (fig. 37). The assumption made in this example is that the value of the α coefficient for a head-dependent (Cauchy-type) boundary changes whenever the boundary condition is located in a different hydraulic-property zone. Note that element sides do not have to be connected to one another in order to be grouped in the same boundary-condition zone (see boundary-condition zone 1 in fig. 37).

Inputs for boundary-condition zones consist of the boundary-zone number, number of element sides contained in the zone, and an indicator variable for the type of zone input (described below). These inputs are followed by values for the α and q_B terms, number of the boundary side, node numbers, and boundary heads. Details of these inputs are given in the section "Input Instructions," as the type of zone input determines the sequence of the other inputs. The zone number and number of element sides are represented in MODFE, respectively, by the program variables KZ and NOS. The number of the boundary side, node numbers, and boundary heads are represented, respectively, by the program variables J; KQB(J) and LQB(J); and, HK(J) and HL(J), as described in the sections "Specified Flux" and "Head-Dependent (Cauchy-Type) Flux."

For each boundary-condition zone, the user has the option of inputting either one value for α and q_B, which will be applied to all boundary sides in the zone, or distinct values for each boundary side. An indicator variable, represented in MODFE as IZIN, is evaluated to determine which type of zone input for α and q_B is used. A value of one (1) for IZIN causes the same value of α and q_B to be applied to all boundary sides in the zone; a value of zero (IZIN = 0 or blank) causes distinct values for α and q_B to be applied to each boundary side in the zone.

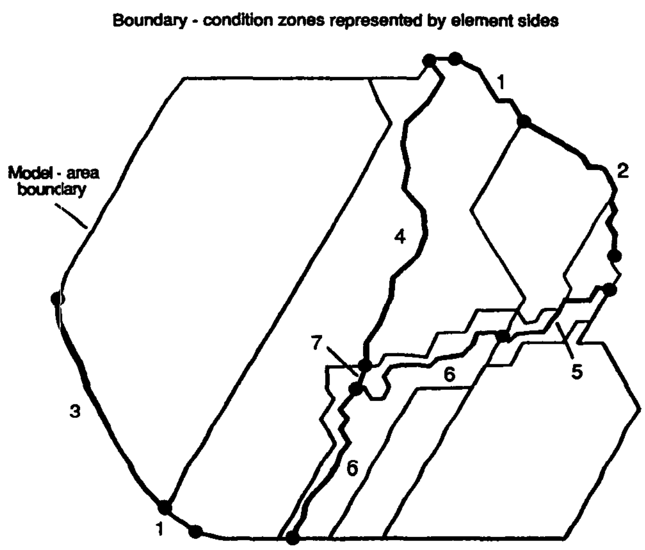


Figure 37.—Areal representation of simulated aquifer showing changes in boundary-condition zones corresponding to changes in hydraulic-property zones.

Table 2.—Data input for nonlinear steady vertical-leakage zones and element incidences

A. **Data defining nonlinear steady vertical-leakage zones for example aquifer problem**

L	NBE	NO	VNCF
1	156	72	5.E-5
2	343	45	1.E-3
3	702	30	3.13E-6

B. **Element incidences for finite-element mesh in example aquifer problem**

Ele. No.	Incidences				
1	30	14	31	55	
2	14	4	15	31	
3	4	1	5	0	
4	15	5	8	20	
5	17	18	6	16	
.	.	.	.	.	
.	.	.	.	.	
.	.	.	.	.	
154	18	19	7	0	
155	16	6	4	14	
156	6	7	1	0	NBE=156
157	2	8	5	1	NO=72
160	97	68	100	138	
.	.	.	.	.	Incidences for
.	.	.	.	.	nonlinear steady
.	.	.	.	.	vertical leakage
					zone 1
226	505	506	427	426	(72 elements)
227	580	581	506	505	
228	645	646	581	580	
229	700	701	646	645	
230	756	757	701	700	
.	.	.	.	.	
.	.	.	.	.	
.	.	.	.	.	
341	814	815	757	756	
342	874	875	815	814	
343	932	933	875	874	NBE=343
344	991	992	933	932	NO=45
345	953	954	992	991	
.	.	.	.	.	Incidences for
.	.	.	.	.	nonlinear steady
.	.	.	.	.	vertical leakage
					zone 2
386	506	507	428	427	(45 elements)
387	581	582	507	506	
388	646	647	582	581	

Ele. No.	Incidences				
389	701	702	647	646	
390	757	758	702	701	
.	.	.	.	.	
.	.	.	.	.	
.	.	.	.	.	
700	875	876	816	815	
701	933	934	876	875	
702	992	993	934	933	NBE=702
703	876	877	817	816	NO=30
704	934	935	877	876	
.	.	.	.	.	Incidences for
.	.	.	.	.	nonlinear steady
.	.	.	.	.	vertical leakage
					zone 3
729	993	994	935	934	(30 elements)
730	877	878	818	817	
731	935	936	878	877	
732	994	995	936	935	
733	878	879	819	818	
734	936	937	879	878	
735	995	996	937	936	
.	.	.	.	.	
.	.	.	.	.	
.	.	.	.	.	

Details of inputs for boundary-condition zones are discussed below and are given in table 3. The first line of input defines, respectively, the values for the zone number (= 1), number of boundary sides (= 9), and the indicator IZIN for the type of input for α and q_B (= 1 for applying the same value to all boundary sides in the zone). The second line gives the values for α (0.05) and q_B (0.). Program variables ALPHZ and QBNZ are used to represent input values for α and q_B, respectively, which are applied to all boundary sides in zone KZ. These inputs are followed by nine lines of input that, for each line, define the number of the boundary side, node numbers on the side, and boundary heads, respectively. The inputs for boundary-

Table 3.—Data input and program variables for boundary-condition zones

A. Boundary-condition zone 1

Variables:	KZ	NOS	IZIN	
Input:	1	9	1	

Variables:	ALPHZ	QBNZ
Input:	0.05	0.

	J	KQB(J)	LQB(J)	HK(J)	HL(J)
Variables:	J	KQB(J)	LQB(J)	HK(J)	HL(J)
Input:	1	102	88	176.	175.
	2	88	97	175.	174.
	3	97	92	174.	172.
	4	92	86	172.	170.
	5	86	80	170.	168.
	6	80	74	168.	165.
	7	74	69	165.	163.
	8	207	156	137.	135.
	9	156	113	135.	134.

B. Boundary-condition zone 2

Variables:	KZ	NOS	IZIN
Input:	2	16	0

Variables:	J	KQB(J)	LQB(J)	ALPH(J)	QBND(J)	HK(J)	HL(J)
Input:	10	113	78	24.21	0.	163.	163.
	11	78	49	28.46		163.	163.
	12	49	26	32.32		163.	164.
	13	26	11	36.57		164.	165.
	14	11	2	41.25		165.	165.
	15	2	5	45.62		165.	166.
	16	5	15	47.80		166.	167.
	17	15	14	53.60		167.	168.
	18	14	16	55.37		168.	168.
	19	16	12	53.25		168.	167.
	20	12	27	51.80		167.	166.
	21	27	50	50.64		166.	164.
	22	50	77	48.90		164.	164.
	23	77	48	47.30		164.	163.
	24	48	25	46.60		163.	162.
	25	26	80	45.20	0.	162.	160.

condition zone 2 begin with values for KZ (= 2), NOS (= 16), and IZIN (= 0). The zero value for IZIN indicates that distinct values for α and q_B will be input for each boundary side in zone 2. These inputs are made on the same line as inputs for the boundary-side number, nodes defining the side, and boundary heads. Program variables ALPH(J) and QBND(J) are used to represent α and q_B, respectively, for these inputs to boundary-side J. Note that for the example-aquifer problem, QBND(J) is set to zero or left blank.

Input Instructions

Inputs to MODFE follow a sequential order according to the Input-Type number and particular version of the main program that is used. Because the user can create versions containing only the simulation capabilities that are pertinent to the aquifer problem to be solved, all inputs listed here may not be required for a particular version of MODFE. Inputs are omitted if they correspond to simulation capabilities that are not contained in the version of MODFE that has been created for the aquifer problem. Specific instructions are given in this section about input types that can be omitted when using certain versions of MODFE, and about input types that are required for all versions. Additional information about inputs for a particular hydrologic feature is given in the corresponding sections preceding the input instructions and in the section "Examples of Model Input." The versions of MODFE are listed in tables 4–6, and program structures for these versions are given in the section "Program Structures and Lists of Main Programs," in Torak (1993).

Table 4.—Linear versions of MODular Finite-Element model (MODFE) and simulation capabilities

<table>
<tr><th colspan="3">Simulation capabilities of linear versions of MODFE</th></tr>
<tr><td>Nonhomogeneous, anisotropic flow having changing directions of anisotropy within model region

Steady vertical leakage (no storage effects)

Point and areally distributed sources and sinks

Specified head (Dirichlet), specified flux (Neumann), and head-dependent (Cauchy-type) boundary conditions</td><td colspan="2">Axisymmetric (radial) flow

Zoned input of hydraulic properties and boundary conditions

Nonsteady-state or steady-state conditions

Vertical cross sections

Changing stresses and boundary conditions with time</td></tr>
<tr><td rowspan="2">Simulation options</td><td colspan="2">Solver options</td></tr>
<tr><td>Direct, symmetric-Doolitle method</td><td>Iterative, MICCG method</td></tr>
<tr><td>Steady vertical leakage (no storage effects)</td><td>LMFE1</td><td>LMFE2</td></tr>
<tr><td>Vertical leakage having storage effects (transient leakage)</td><td>LMFE3</td><td>LMFE4</td></tr>
</table>

Table 5.—Nonlinear versions of MODular Finite-Element model (MODFE) and simulation capabilities

Simulation capabilities of nonlinear versions of MODFE	
Nonhomogeneous, anisotropic flow having changing directions of anisotropy within model region	Zoned input of hydraulic properties and boundary conditions
Steady vertical leakage (no storage effects)	Nonsteady-state conditions
Point and areally distributed sources and sinks	Unconfined (water-table) conditions
Specified head (Dirichlet), specified flux (Neumann), and head-dependent (Cauchy-type) boundary conditions	Partial drying and resaturation of a water-table aquifer
Axisymmetric (radial) flow	Conversion between confined- and unconfined-aquifer conditions
	Change stresses and boundary conditions with time

Simulation options	Solver options	
	Direct, triangular-decomposition method	Iterative, MICCG method
Steady vertical leakage (no storage effects)	NLMFE1	NLMFE2
Vertical leakage having storage effects (transient leakage)	NLMFE3	NLMFE4
Nonlinear steady vertical leakage	NLMFE5	NLMFE6
Nonlinear head-dependent (Cauchy-type) boundaries	NLMFE7	NLMFE8

Table 6.—Nonlinear steady-state versions of MODular Finite-Element model (MODFE) and simulation capabilities

<table>
<tr><th colspan="3">Simulation capabilities of nonlinear versions of MODFE</th></tr>
<tr><td colspan="2">Nonhomogeneous, anisotropic flow having changing directions of anisotropy within model region

Steady vertical leakage (no storage effects)

Point and areally distributed sources and sinks

Specified head (Dirichlet), Specified flux (Neumann), and head-dependent (Cauchy-type) boundary conditions

Axisymmetric (radial) flow</td><td>Zoned input of hydraulic properties and boundary conditions

Steady-state conditions

Unconfined (water-table) conditions

Partial drying and resaturation of a water-table aquifer

Conversion between confined- and unconfined-aquifer conditions</td></tr>
<tr><td rowspan="2">Simulation options</td><td colspan="2">Solver options</td></tr>
<tr><td>Iterative, MICCG method</td><td>Direct, triangular-decomposition method</td></tr>
<tr><td>Water-table conditions only</td><td>NSSFE1</td><td>NSSFE2</td></tr>
<tr><td>Nonlinear steady vertical leakage</td><td>NSSFE3</td><td>NSSFE4</td></tr>
<tr><td>Nonlinear head-dependent (Cauchy-type) boundaries</td><td>NSSFE5</td><td>NSSFE6</td></tr>
</table>

Input-Types 1 and 2: Title and Problem Specifications

Required for all versions of MODFE. Replace MBW by NIT, the maximum number of iterations, if the conjugate-gradient method, MICCG, is used for solution.

Input Type	Number of Records	Format	Program Variable	Definition
1	3	20A4	TITLE	Title of simulation problem.
2	1	16I5	NELS	Number of triangular elements or element pairs for which element incidences will be input (see section "Combined-Element Incidences").
			NNDS	Number of nodes.
			MXSTPS	Maximum number of time steps in any stress period. (Number of time steps in first stress period is input later as NTMP.)
			NPER	Number of stress periods. See section "Selecting Stress Periods and Time-Step Sizes" for establishing stress periods.
			NZNS	Number of aquifer-property zones.
			NWELS	Initial number of point sources or sinks (wells).
			NQBND	Total number of element sides on Cauchy-type boundaries, includes specified-flux and head-dependent (Cauchy-type) flux boundaries.
			NBCZ	Number of zones for Cauchy-type boundaries (see section "Grouping Element Sides into Zones").
			NHDS	Number of specified-head nodes.
			MBWC	Maximum condensed-matrix bandwidth (see section "Node Numbering and Determining Bandwidth").
			MBW	*Reduced-matrix bandwidth.*

Input-Type 2A is required for the iterative, MICCG method of solution (see section "Iterative-- Modified Incomplete-Cholesky Conjugate Gradient").

OMIT INPUT-TYPE 2A IF DIRECT-METHOD OF TRIANGULAR DECOMPOSITION IS USED FOR SOLUTION (SUBROUTINE BAND)

2A	1	F10.0	TOL	Closure tolerance for conjugate-gradient solution.

Input-Type 2B is required for simulating nonlinear steady-state conditions.

OMIT INPUT TYPE 2B FOR SIMULATION OF NONSTEADY-STATE CONDITIONS AND LINEAR STEADY-STATE CONDITIONS (See sections "Linear Conditions and Nonlinear Conditions.")

2B	1	I5, 2F10.0	NITSW	Maximum number of water-table iterations.
			TOLSW	Closure tolerance for steady state.
			DSMX	Maximum allowable displacement, or head change during a water-table iteration.

Input-Type 2C is required for simulating nonlinear head-dependent fluxes. (See sections "Nonlinear Head-Dependent Flux," "Cauchy Type," and "Point Sinks.")

OMIT INPUT-TYPE 2C IF NONLINEAR HEAD-DEPENDENT FLUXES ARE NOT SIMULATED

2C	1	16I5	NBNC	Number of element sides on nonlinear head-dependent (Cauchy-type) flux boundaries.
			NLCZ	Number of zones for nonlinear head-dependent (Cauchy-type) boundaries.
			NPNB	Number of nonlinear point sinks.

Input-Type 2D is required for simulating nonlinear steady vertical leakage (see subsection "Steady Vertical Leakage and Evapotranspiration" of section "Nonlinear Head-Dependent Flux").

OMIT INPUT-TYPE 2D IF NONLINEAR STEADY VERTICAL LEAKAGE IS NOT SIMULATED

2D	1	16I5	NVNZ	Number of nonlinear steady vertical-leakage zones.

Input-Type 2E is required for simulating transient leakage (see section "Vertical Leakage of Water Stored Elastically in a Confining Bed").

OMIT INPUT-TYPE 2E IF TRANSIENT LEAKAGE IS NOT SIMULATED

2E	1	2I5	NCBZ	Number of transient-leakage zones.
			MCBN	Maximum number of nodes where transient-leakage is simulated.

Input-Type 3: Indicator Variables for Axisymmetric-Cylindrical Flow, Scaled Coordinates, and Steady-State Simulations

Required for all versions of MODFE.

Input Type	Number of Records	Format	Program Variable	Definition
3	1	16I5	IRAD	Indicator for coordinate system: = 0 for Cartesian (x-y), = 1 for axisymmetric cylindrical.
			IUNIT	Indicator for scaling units of nodal coordinates: = 0 for no scaling, = 1 for scaling (see definition of SCALE below).
			ISTD	Indicator for steady-state simulations: = 0 for nonsteady state, = 1 for steady state.

Input-Type 4: Title and Scale Factor for Scaling Map Units into Field Units

OMIT INPUT-TYPE 4 IF IUNIT = 0

Input Type	Number of Records	Format	Program Variable	Definition
4	1	20A4	TITLE	Title for scaling factor. Example, "1 inch (map unit) equals 1,000 feet (field unit)."
	1	8F10.0	SCALE	Scale (multiplication) factor for converting map units of length to field units. Used to scale nodal coordinates, lengths, and areas. In above example, SCALE = 1000.

Input-Type 5: Suppress Printout of Initial Conditions

Required for all versions of MODFE.

Input Type	Number of Records	Format	Program Variable	Definition
5	1	16I5	IPXY	Indicator for nodal coordinates: = 0 to print coordinates, = 1 to suppress printout.
			IPH	Indicator for aquifer heads: = 0 to print initial heads, = 1 to suppress printout.
			IPHR	Indicator for source-bed heads: = 0 to print source-bed heads, = 1 to suppress printout.
			IPQW	Indicator for point sources and sinks: = 0 to print sources and sinks, = 1 to suppress printout.
			IPCB	Indicator for specified-flux and head-dependent (Cauchy-type) flux: = 0 to print boundary input, = 1 to suppress printout.
			IPHB	Indicator for initial values on specified-head boundaries: = 0 to print specified heads, = 1 to suppress printout.
			IPND	Indicator for element incidences (node numbers for each element): = 0 to print incidence list, = 1 to suppress printout.

Input-Type 6: Node Coordinates and Head Data

Required for all versions of MODFE.

Input Type	Number of Records	Format	Program Variable	Definition
6	NNDS	I5, 7F10.0	I	Node number.
			XG(I)	X coordinate of node [length].
			YG(I)	Y coordinate of node [length].
			H(I)	Initial hydraulic head [length].
			HR(I)	Source-bed head [length].

Input-Types 7-9: Boundary Conditions

For steady-state simulations, at least one specified-head boundary, Input-Type 9, or one head-dependent (Cauchy-type) boundary with ALPHZ or ALPH(J) > 0, Input-Type 8, is required to obtain a unique solution. A nonlinear head-dependent (Cauchy-type) boundary with GCZ or GC(I) > 0, Input-Types 13C or 13D may replace Input-Type 8 for this requirement.

Input-Type 7: Point Sources and Sinks

OMIT INPUT-TYPE 7 IF NWELS = 0

Input Type	Number of Records	Format	Program Variable	Definition
7	NWELS	I5, 7F10.0	I	Node number of point source/sink.
			QWEL	Volumetric flow rate [length^3 / time] of point source/sink.

Input-Type 8: Specified-Flux and Head-Dependent (Cauchy-Type) Flux

Input-Type 8A is followed by either Input-Type 8B or Input-Type 8C for each zone (see section "Grouping Element Sides into Zones" for details).

OMIT INPUT-TYPES 8A-8C IF NQBND = 0

Input Type	Number of Records	Format	Program Variable	Definition
8A	NBCZ	16I5	KZ	Zone number for specified flux or head-dependent (Cauchy-type) flux boundary.
			NOS	Number of element sides in zone.
			IZIN	Indicator for zone input of α and/or q_B: = 0 for inputting unique values for each side in zone, = 1 for inputting one value for all sides in zone.

Input-Type 8B follows Input-Type 8A for each zone.

OMIT INPUT-TYPE 8B IF IZIN = 0

Input Type	Number of Records	Format	Program Variable	Definition
8B	NBCZ	8F10.0	ALPHZ	α term for head-dependent (Cauchy-type) boundary [length/time]. See section "Head-Dependent (Cauchy-Type) Flux" for applications.
			QBNZ	q_B term for specified-flux boundary [length^2/time]. See section "Specified Flux" for applications.
	NOS	3I5, 4F10.0	J	Number of the boundary side.
			KQB(J)	Node k of element side J on boundary.
			LQB(J)	Node l of element side J on boundary.
			HK(J)	Boundary or external head, H_B [length], at node k on boundary (see fig. 16).
			HL(J)	Boundary or external head, H_B [length], at node l on boundary (see fig. 16).

Input-Type 8C follows Input-Type 8A for each zone.

OMIT INPUT-TYPE 8C IF IZIN = 1

Input Type	Number of Records	Format	Program Variable	Definition
8C	NOS	3I5, 4F10.0	J	Number of the boundary side (see figs. 15-18, 20).
			KQB(J)	Node k of element side J on boundary.
			LQB(J)	Node l of element side J on boundary.
			ALPH(J)	α term for head-dependent Cauchy-type boundary [length/time]. See section "Head-Dependent (Cauchy-Type) Flux" for applications.
			QBND(J)	q_B term for specified-flux boundary [length^2/time]. See section "Specified Flux" for applications.
			HK(J)	Boundary or external head, H_B [length], at node k on boundary (see fig. 16).
			HL(J)	Boundary or external head, H_B [length], at node l on boundary (see fig. 16).

Input-Type 9: Specified Heads

OMIT INPUT-TYPE 9 IF NHDS = 0

Input Type	Number of Records	Format	Program Variable	Definition
9	NHDS	I5, 7F10.0	J	Node number of specified-head boundary.
			HB	Initial value of specified head at node J [length].

Input-Types 10 and 11: Hydraulic-Property Values and Element Incidences

Required for all versions of MODFE. Enter input-types 10 and 11 together for each hydraulic-property zone, KZ (see section "Combined-Element Incidences" and "Grouping Elements into Zones" for details).

Input Type	Number of Records	Format	Program Variable	Definition
10	NZNS	2I5, 6F10.0	KZ	Hydraulic-property-zone number.
			NO	Number of elements or element pairs in zone.
			XTR	X transmissivity, T_{xx} [length^2/time], for confined flow or x hydraulic conductivity, K_{xx} [length/time], for unconfined flow in areal dimensions; radial hydraulic conductivity, K_{rr}, [length/time], for axisymmetric (radial) flow, or horizontal hydraulic conductivity K_{xx} or K_{yy} for cross-sectional flow.
			YTR	Y transmissivity, T_{yy} [length^2/time], for confined flow or y hydraulic conductivity, K_{yy} [length/time], for unconfined flow in areal dimensions; vertical hydraulic conductivity K_{zz}, [length/time], for radial or cross-sectional flow.
			ANG	Rotation angle (in degrees) for transforming global x-y coordinates to the local $\bar{x}$-$\bar{y}$ system for varying directions of anisotropy.
			VLC	Hydraulic conductance (vertical hydraulic conductivity divided by thickness) of confining bed [time^{-1}].
			STR	Storage coefficient [dimensionless] for confined conditions; specific yield [dimensionless] for unconfined conditions without conversion between confined and unconfined aquifer conditions, or specific storage [length^{-1}] for cross-sectional or radial flow
			QD	Unit rate of areally distributed stress [length/time].
11	NO	16I5	IEL	Element number.
			ND(I)	Element incidences. Four values required for each element or element pair. Element pair is counted as one element and is divided along first and third entries of incidences (see section "Combined-Element Incidences" for details.

Input-Type 12: Unconfined (Water-Table) Conditions

Required for the following nonlinear, nonsteady-state simulations: water-table conditions, conversion between confined and unconfined aquifer conditions, drying and resaturation of aquifer material, and nonlinear head-dependent fluxes. See appropriate sections pertaining to these hydrologic conditions for information on additional inputs. The value for * appearing in the "Number of Records" column of Input-Types 12B and 12C is computed as (NNDS+7)/8 by using integer math.

OMIT INPUT-TYPE 12 FOR SIMULATION OF CONFINED CONDITIONS

OMIT INPUT-TYPE 12D FOR NONLINEAR, STEADY-STATE SIMULATIONS

Input Type	Number of Records	Format	Program Variable	Definition
12A	1	16I5	IPTK	Indicator to suppress printout of aquifer thickness at each node: = 0 to print thickness values, = 1 to suppress printout.
			IPTP	Indicator to suppress printout of altitude of top of aquifer or bottom of overlying confining bed at each node: = 0 to print altitude values, = 1 to suppress printout.
12B	*	8F10.0	THK(I)	Nodal value of aquifer thickness [length].
12C	*	8F10.0	TOP(I)	Nodal value for altitude of top of water-table aquifer or bottom of overlying confining bed [length].
12D	NZNS	2I5, F10.0	KZ	Zone number for specific yield.
			NO	Number of elements in zone.
			SY	Specific yield [dimensionless].

Input-Types 13-15: Nonlinear Head-Dependent Flux

Input requirements vary depending on the program structure of MODFE and type of nonlinear conditions that are simulated. For details about inputs, refer to instructions at the beginning of each Input Type and in corresponding sections of this report. See sections "Nonlinear Head-Dependent Flux" and "Program Structures and Lists of Main Programs" in Torak (1992) for programming details.

OMIT INPUT-TYPES 13-15 FOR SIMULATION OF LINEAR-FLOW CONDITIONS

Input-Type 13: Nonlinear Head-Dependent (Cauchy-Type) Flux and Point Sinks

Input-Type 13A required for simulating nonlinear head-dependent (Cauchy-type) flux and nonlinear point sinks.

Input Type	Number of Records	Format	Program Variable	Definition
13A	1	2I5	IPNC	Indicator to suppress printout of input for nonlinear head-dependent (Cauchy-type) boundaries: = 0 to print boundary values, = 1 to suppress printout.
			IPNP	Indicator to suppress printout of input for nonlinear point-sink boundaries: = 0 to print boundary values, = 1 to suppress printout.

Input-Type 13B: Nonlinear Head-Dependent (Cauchy-Type) Flux

Input-Type 13B is followed by either Input-Type 13C or Input-Type 13D for each zone (see section "Grouping Element Sides into Zones" for details).

OMIT INPUT-TYPES 13B-13D IF NBNC = 0

Input Type	Number of Records	Format	Program Variable	Definition
13B	NLCZ	16I5	KZ	Zone number for nonlinear head-dependent (Cauchy-type) boundary.
			NOS	Number of sides in zone.
			IZIN	Indicator for zone input of α terms: = 0 for inputting unique values for each side in zone, = 1 for inputting one value for all sides in zone.

Input-Type 13C: Nonlinear Head-Dependent (Cauchy-Type) Flux

Input-Type 13C follows Input-Type 13B for each zone. The value of GCZ for zone KZ is followed, on separate lines, by inputs for the boundary-side number, J; nodes on the boundary side, KR(J) and LR(J); and boundary and controlling heads, in a manner similar to Input-Type 8B (see section "Cauchy Type" for applications of this boundary condition and descriptions of inputs).

OMIT INPUT-TYPE 13C IF IZIN = 0

Input Type	Number of Records	Format	Program Variable	Definition
13C	NLCZ	F10.0	GCZ	α term for nonlinear head-dependent (Cauchy-type) boundary zone [length/time].
	NOS	3I5, 5F10.0	J	Number of the boundary (element side) in zone KZ.
			KR(J)	Node k of element side J on boundary.
			LR(J)	Node l of element side J on boundary.
			HRK(J)	Boundary or external head H_r at node k on boundary [length].
			HRL(J)	Boundary or external head H_r at node l on boundary [length].
			ZRK(J)	Controlling head or altitude z_r at node k on boundary [length].
			ZRL(J)	Controlling head or altitude z_r at node l on boundary [length].

Input-Type 13D: Nonlinear Head-Dependent (Cauchy-Type) Flux

Input-Type 13D follows Input-Type 13B for each zone. See section "Cauchy Type" for applications of this boundary condition and descriptions of inputs.

OMIT INPUT-TYPE 13D IF IZIN = 1

Input Type	Number of Records	Format	Program Variable	Definition
13D	NOS	3I5, 5F10.0	J	Number of boundary (element side) in zone KZ.
			KR(J)	Node k of element side J on boundary.
			LR(J)	Node l of element side J on boundary.
			GC(J)	α term for nonlinear head-dependent (Cauchy-type) boundary side [length/time].
			HRK(J)	Boundary or external head H_r at node k on boundary [length].
			HRL(J)	Boundary or external head H_r at node l on boundary [length].
			ZRK(J)	Controlling head or altitude z_r at node k on boundary [length].
			ZRL(J)	Controlling head or altitude z_r at node l on boundary [length].

Input-Type 14: Nonlinear Point Sinks

See section "Point Sinks" for applications of this boundary condition and descriptions of inputs.

OMIT INPUT-TYPE 14 IF NPNB = 0

Input Type	Number of Records	Format	Program Variable	Definition
14	NPNB	2I5, 2F10.0	I	Number of the point boundary.
			KP(I)	Node number at point boundary.
			GCP	Discharge coefficient Cp_l for point boundary [length^2/time].
			ZP(I)	Reference altitude, z_p, for point sink [length].

Input-Type 15: Nonlinear Steady Vertical Leakage

Input-Type 15A is followed by entries of Input-Type 15B for all nonlinear steady vertical-leakage zones. One entry of Input-Type 15A is required; the option to print or suppress printout of zone inputs and nodal values of HS is applied to all nonlinear steady vertical-leakage zones and to all nodes. The value for * appearing in the "Number of Records" column of Input-Type 15C is computed as (NNDS+7)/8 by using integer math. See subsection "Steady Vertical Leakage" under "Nonlinear Head-Dependent Flux" and section "Zones for Nonlinear Steady Vertical Leakage, Transient Leakage, and Specific Yield" for applications of this boundary condition and descriptions of inputs.

OMIT IF NONLINEAR STEADY VERTICAL LEAKAGE IS NOT SIMULATED

Input Type	Number of Records	Format	Program Variable	Definition
15A	1	3I5	IPNV	Indicator to suppress printout of zone input for all nonlinear steady vertical-leakage zones: = 0 to print zone values, = 1 to suppress printout.
			IPHS	Indicator to suppress printout of controlling head or altitude, HS, by node: = 0 to print HS, = 1 to suppress printout.
15B	NVNZ	3I5, F10.0	L	Zone number for nonlinear leakage.
			NBE	Beginning element number in zone.
			NO	Number of elements in zone.
			VNCF	Conductance terms, R_a or R_e [time^{-1}].
15C	*	8F10.0	HS(I)	Controlling head, H_a, or altitude, z_e or z_t, at node I.

Input-Type 16: Transient-Leakage Approximation

See section "Zones for Nonlinear Steady-Vertical Leakage, Transient Leakage, and Specific Yield" for descriptions about establishing zones and program variables.

OMIT INPUT-TYPE 16 IF TRANSIENT LEAKAGE IS NOT SIMULATED

Input Type	Number of Records	Format	Program Variable	Definition
16	NCBZ	3I5, 2F10.0	L	Transient-leakage-zone number.
			NBE	Beginning element number in zone.
			NO	Number of elements in zone.
			VCON	Vertical hydraulic conductivity of confining bed [length/time].
			SPST	Specific storage of confining bed [length $^{-1}$].

Input-Types 17-25: Stress-Period Inputs

Begin inputs for each stress period with Input-Type 17, followed by optional Input-Types 18-25 for each time step when changes in stresses or boundary conditions are to be made. The number of inputs of Input-Type 17 followed by the appropriate entries of Input-Types 18-25 is given by the value entered for the number of stress periods, NPER. Thus, the "1" appearing in the "Number of Records" column for Input-Type 17A implies that the Input Type is used once for a stress period. The value for * appearing in the "Number of Records" column of Input-Type 17B is computed as (NTMP+7)/8 by using integer math (see sections "Changing Stresses and Boundary Conditions with Time" and "Examples of Model Input" for details).

OMIT INPUT TYPES 17-25 FOR NONLINEAR STEADY-STATE SIMULATIONS

Input-Type 17A: Indicators for Time Varying Stresses and Boundary Conditions

Required for nonsteady-state and linear steady-state versions of MODFE.

Input Type	Number of Records	Format	Program Variable	Definition
17A	1	16I5	NTMP	Number of time steps in stress period. Exception: set to zero (0) to use time steps from previous stress period; set to one (1) for linear steady-state simulations.
			NWCH	Time-step number when point sources and sinks are changed.
			NQCH	Time-step number when areally distributed sources and sinks are changed.
			NHRCH	Time-step number when source-bed heads, H, are changed for simulating steady vertical leakage (no transient leakage from confining bed).
			NBQCH	Time-step number when specified flux or head-dependent (Cauchy-type) flux boundaries are changed.
			NHCH	Time-step number when values for specified-head boundaries are changed.
			NCBCH	Time-step number when source-bed heads, H, are changed for simulating transient leakage.
			NVNCH	Time-step number when controlling heads or altitudes, H_a, z_e, or z_t, for nonlinear steady vertical leakage are changed.
			NGNCH	Time-step number when boundary or external heads, H_r, on nonlinear head-dependent (Cauchy-type) boundaries are changed.

Input-Type 17B: Time Steps

Required for first stress period of nonsteady-state versions of MODFE and subsequent stress periods if NTMP > 0 (see section "Selecting Stress Periods and Time-Step Sizes" for details). Required for linear steady-state versions of MODFE (see definition of DELT(I) below).

OMIT FOR STRESS PERIODS SUBSEQUENT TO STRESS PERIOD 1 IF NTMP = 0

Input Type	Number of Records	Format	Program Variable	Definition
17B	*	8F10.0	DELT(I)	Time-step sizes [time]; any units consistent with hydraulic properties. Exception: set to one (1.) for linear steady-state simulations.

Input-Types 18-25: Changing Stresses and Boundary Conditions with Time

Required on the time step and stress period when changes are to occur. The "1" listed in the "Number of Records" column for Input-Types 18A, 19A, ..., 25A indicates that these inputs are entered once on the time step in which the corresponding change in stress or boundary condition is made. Each of the "A" Input Types is followed by inputs of the corresponding "B" types. Some changes are implemented by inputs on two successive time steps (see section "Changing Stresses and Boundary Conditions with Time" and "Examples of Model Input" for details).

Input-Type 18: Changes to Point Sources and Sinks

OMIT FOR TIME STEPS IN WHICH POINT SOURCES AND SINKS ARE UNCHANGED FROM PREVIOUS VALUES

Input Type	Number of Records	Format	Program Variable	Definition
18A	1	16I5	N	Number of point sources and sinks to be changed on this time step.
			NWCH	Time-step number for additional changes to point sources and sinks during present stress period.
18B	N	I5, 4F10.0	J	Node number of point source or sink to be changed.
			QOLD	Old value of stress [length ^{3}/ time] to be changed.
			QNEW	New value of stress [length ^{3}/ time] to replace old value.

Input-Type 19: Changes to Areally Distributed Sources and Sinks

OMIT FOR TIME STEPS IN WHICH AREALLY DISTRIBUTED SOURCES AND SINKS ARE UNCHANGED FROM PREVIOUS VALUES

Input Type	Number of Records	Format	Program Variable	Definition
19A	1	16I5	N	Number of zones for areally distributed sources and sinks that are changed on this time step.
			NQCH	Time-step number for additional changes to areally distributed sources and sinks during present stress period.
19B	N	3I5, 2F10.0	L	Zone number for areally distributed source or sink to be changed.
			NBE	Beginning element number in zone.
			NO	Number of elements in zone.
			QOLD	Old value of unit-areal stress [length/time] to be changed.
			QNEW	New value of unit-areal stress [length/time] to replace old value.

Input-Type 20: Changes to Source-Bed Heads for Steady Vertical Leakage

OMIT FOR TIME STEPS IN WHICH SOURCE-BED HEADS FOR STEADY VERTICAL LEAKAGE ARE UNCHANGED FROM PREVIOUS VALUES

Input Type	Number of Records	Format	Program Variable	Definition
20A	1	16I5	N	Number of nodes for source-bed heads that are changed on this time step.
			NHRCH	Time-step number for additional changes to source-bed heads during present stress period.
20B	N	I5, F10.0	J	Node number of source-bed head to be changed.
			HR(J)	New value of source-bed head [length].

Input-Type 21: Changes to Specified Flux or Boundary Head on Cauchy-Type Boundaries

OMIT INPUT-TYPE 21 IF CAUCHY-TYPE BOUNDARIES ARE UNCHANGED FROM PREVIOUS VALUES

Input Type	Number of Records	Format	Program Variable	Definition
21A	1	16I5	N	Number of boundary sides to be changed on this time step.
			NBQCH	Time-step number for making additional changes to Cauchy-type boundaries in present stress period.
21B	N	I5, 4F10.0	J	Number of the boundary side.
			QNEW	New value of unit discharge, q_B [length ^{2}/time], to replace old value.
			HK(J)	New value for boundary or external head, H_B [length], at node k on boundary side J (see fig. 16).
			HL(J)	New value for boundary or external head, H_B [length], at node l on boundary side J (see fig. 16).

Input-Type 22: Changes to Specified-Head Boundaries

OMIT INPUT-TYPE 22 IF SPECIFIED HEADS ARE UNCHANGED FROM PREVIOUS TIME STEP

Input Type	Number of Records	Format	Program Variable	Definition
22A	1	16I5	N	Number of specified-head boundaries (nodes) to be changed on this time step.
			NHCH	Time-step number for additional changes to specified-head boundaries during present stress period.
22B	N	I5, 4F10.0	J	Node number of the specified-head boundary to be changed.
			HB	New value of the specified head for node J [length].

Input-Type 23: Changes to Source-Bed Heads for Transient Leakage

OMIT INPUT-TYPE 23 IF SOURCE-BED HEADS FOR TRANSIENT LEAKAGE ARE UNCHANGED FROM PREVIOUS TIME STEP

Input Type	Number of Records	Format	Program Variable	Definition
23A	1	2I5	N	Number of nodes where source-bed heads are to be changed on this time step.
			NCBCH	Time-step number for additional changes to source-bed heads during present stress period.
23B	N	I5, F10.0	J	Node number of the source-bed head to be changed.
			HR(J)	New value of the source-bed head for node J [length].

Input-Type 24: Changes to Boundary or External Heads on Nonlinear Head-Dependent (Cauchy-Type) Boundaries

OMIT INPUT-TYPE 24 IF BOUNDARY OR EXTERNAL HEADS ARE UNCHANGED FROM PREVIOUS TIME STEP

Input Type	Number of Records	Format	Program Variable	Definition
24A	1	16I5	N	Number of boundary sides to be changed on this time step.
			NGNCH	Time-step number for additional changes to boundary or external heads during present stress period.
24B	N	I5, 2F10.0	J	Number of the nonlinear boundary side to be changed.
			HRK(J)	New value of the boundary head, H_r, for node k on side J of nonlinear boundary [length].
			HRL(J)	New value of the boundary head, H_r, for node l on side J of nonlinear boundary [length].

Input-Type 25: Changes to Controlling Heads for Nonlinear Steady Vertical Leakage

OMIT INPUT-TYPE 25 IF CONTROLLING HEADS ARE UNCHANGED FROM PREVIOUS TIME STEP

Input Type	Number of Records	Format	Program Variable	Definition
25A	1	2I5	N	Number of nodes where controlling heads are to be changed on this time step.
			NVNCH	Time-step number for additional changes to controlling heads during present stress period.
25B	N	I5, F10.0	J	Node number of the controlling head to be changed.
			HR(J)	New value of the controlling head for node J [length].

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APPENDICES

Appendices

Definition of Input and Output Files

All Input Types are arranged in one file called MODFE.DAT, which is defined and opened by a Fortran statement in the main program. Fortran-unit number 50 is assigned to this file by the "OPEN" statement. The user can change the name of the file in the "OPEN" statement or delete the statement from the program and open the input file by commands to the operating system of the computer. The Fortran-unit number for input (50) is represented in MODFE by the program variable IIN. To change the Fortran-unit number, the user must change the program statements in subroutines INITB or INITCG where the value of IIN is assigned.

Output from MODFE is placed in a file called MODFE.OUT, which is defined and opened by a Fortran statement in the main program. Model output is written to MODFE.OUT by using Fortran-unit number 60. Like the input file, the user can change the name of the output file on the "OPEN" statement, or delete the statement from the program and open the output file by commands to the operating system of the computer. The Fortran-unit number is represented in MODFE by the program variable IOUT, which is assigned the value of 60 in subroutines INITB and INITCG.

Two temporary-storage files are used by MODFE during simulation. These files are used to store terms that form coefficients to the finite-element equations prior to solution (on Fortran unit 55) and to store element areas and incidences (on Fortran unit 56). The Fortran-unit numbers are represented by program variables ITA (=55) and ITB (=56), and are assigned values in subroutines INITB and INITCG. The files are opened by statements in the main program, which can be deleted and replaced by commands to the operating system, if desired.

Examples of Model Input

Examples of input to MODFE that correspond to four, simplified aquifer problems are presented in this section. Output corresponding to these simulations are given in the following section. Structures of the main programs that were used for these simulations are listed in the section "Program Structures and Lists of Main Programs," in Torak (1993).

The first example input corresponds to a simulation of nonsteady-state flow in a confined aquifer (table 7). The finite-element mesh consists of two elements and four nodes (fig. 38). Three stress periods are simulated in which values for areally distributed stress and specified-heads are changed at the beginning of the second and third stress periods. Areally distributed recharge is applied during the second stress period only, and nodes 2 and 3 are specified-head boundaries. Note that the indicators for changing stresses or boundary conditions (Input-Type 17A) are set to zero for the first stress period, and the appropriate indicators are set to 1 for stress-periods 2 and 3. The aquifer problem was solved by using the linear version of MODFE termed LMFE1.

The two-element, four-node mesh from the first example (fig. 38) is used in the second example to demonstrate inputs for nonlinear conditions. A surficial (unconfined) aquifer contains a discharge well at node 1, two specified-head boundaries at nodes 2 and 3, and a nonlinear head-dependent (Cauchy-type) boundary along the element side defined by nodes 1 and 4 (table 8). The controlling head, H_r, to the nonlinear boundary condition is changed on the second and third stress periods. Because the aquifer is surficial, values for TOP, Input-Type 12C, represent the altitude of land surface. Also, conversion between confined and unconfined conditions cannot occur; thus, the specific yield (= 0.1) is input for the program variable STR (Input-Type 10) in addition to its input for SY (Input-Type 12D). The nonlinear version of MODFE termed NLMFE8 that uses the iterative, conjugate-gradient (MICCG) method is used for this simulation.

The third example demonstrates input for a steady-state, water-table simulation by using the finite-element mesh shown in figure 38. The aquifer problem contains a nonlinear point sink at node 1, nonlinear steady vertical leakage in both elements, and specified-head boundaries at nodes 2 and 3 (table 9). The nonlinear steady vertical leakage is of the discharge-only type, which is indicated by the input of a negative value for the leakage coefficient, program variable VNCF, as Input-Type 15B. The MICCG method is used to solve the finite-element equations. Inputs that control water-table iterations are made as Input-Type 2B. Note that the closure tolerance (TOL) for MICCG iterations is 0.1 (Input-Type 2A) and the tolerance for water-table iterations (TOLSW) is 0.0001 (Input-Type 2B). By specifying TOL greater than TOLSW, an acceptable solution of hydraulic head is obtained with less computational work than if-

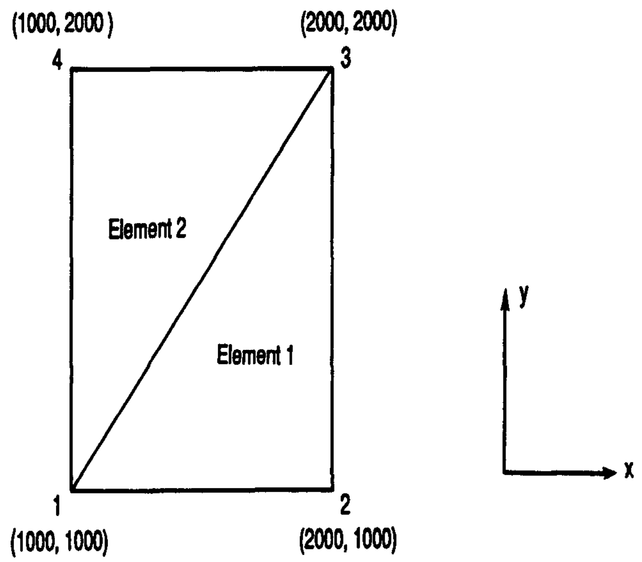

Figure 38.—Two-element, four-node, finite-element mesh used in examples of input to MODular Finite-Element model (MODFE).

TOLSW were greater than TOL (see section "Stopping Criteria" in Cooley (1992) for details). The computational features of two versions of MODFE, NSSFE3 and NSSFE5, are combined to solve the aquifer problem given as example 3.

The fourth example (table 10) lists the input that was used in Cooley (1992) to obtain computed values of hydraulic head which are compared with the analytical solution for conversion between confined and unconfined conditions (Moench and Prickett, 1972). Problem specifications and details of the finite-element discretization in space and time are given by Cooley (1992). In general, the aquifer problem is solved by using 68 triangular elements, 52 nodes, and 44 time steps. A head-dependent (Cauchy-type) boundary is placed at a distance of 32,000 meters from the pumped well to provide inflow to the simulated region from an aquifer that is assumed to be infinite in areal extent. The nonlinear version of MODFE termed NLMFE8 is used to simulate this aquifer problem.

Table 7.—Data input and descriptions for first example problem

Input Type	Description	Input
1:	TITLE	*** EXAMPLE 1. -- TWO-ELEMENT, FOUR-NODE PROBLEM SOLVED WITH LMFE1.
1:	TITLE	*** CONTAINS THREE STRESS PERIODS WITH CHANGES TO SPECIFIED HEADS
1:	TITLE	*** AND AREALLY DISTRIBUTED STRESSES ON EACH STRESS PERIOD.
2:	Problem Specifications	2 4 1 3 1 0 1 1 2 4 2
3:	IRAD IUNIT ISTD	0 1 0
4:	TITLE	** 1 MAP UNIT = 1000 FIELD UNITS (FT); TIME IN SECONDS **
4:	SCALE	1000.
5:	Suppress Printout	0 0 0 0 0 0 0
6:	I XG(I) YG(I) H(I) HR(I)	1 1. 1. 100. 0.
6:	I XG(I) YG(I) H(I) HR(I)	2 2. 1. 100. 0.
6:	I XG(I) YG(I) H(I) HR(I)	3 2. 2. 100. 0.
6:	I XG(I) YG(I) H(I) HR(I)	4 1. 2. 100. 0.
8A:	KZ NOS IZIN	1 1 1
8B:	ALPHZ QBNZ	0. .002
8B:	J KQB(J) LQB(J) HK(J) HL(J)	1 1 4 0. 0.
9:	J HB	2 90.
9:	J HB	3 90.
10:	Hydraulic Properties	1 2 1E-3 1E-3 0. 0. .001 0.
11:	IEL ND(I) I=1,4	1 1 2 3 0
11:	IEL ND(I) I=5,8	2 1 3 4 0
17A:	Stress-Period Indicators	1 0 0 0 0 0 0 0 0
17B:	DELT(1)	5000.
17A:	Stress-Period Indicators	1 0 1 0 0 1 0 0 0
17B:	DELT(1)	2000.
19A:	N NQCH	1 0
19B:	L NBE NO QOLD QNEW	1 1 2 0.0 1.E-6
22A:	N NHCH	2 0
22B:	J HB	2 110.
22B:	J HB	3 110.
17A:	Stress-Period Indicators	1 0 1 0 0 1 0 0 0
17B:	DELT(1)	3000.
19A:	N NQCH	1 0
19B:	L NBE NO QOLD QNEW	1 1 2 1.E-6 0.0
22A:	N NHCH	2 0
22B:	J HB	2 100.
22B:	J HB	3 100.

Table 8.—Data input and descriptions for second example problem

Input Type	Description	Input
1:	TITLE	*** EXAMPLE 2. -- TWO-ELEMENT, FOUR-NODE PROBLEM SIMULATING WATER-TABLE CONDI-
1:	TITLE	*** TIONS. THREE STRESS PERIODS SHOW CHANGES TO CONTROLLING HEADS AT NONLIN-
1:	TITLE	*** EAR CAUCHY-TYPE BOUNDARIES AND TIME-STEP SIZES. SOLUTION BY MICCG METHOD.
2:	Problem Specifications	1 4 3 3 1 1 0 0 2 4 10
2A:	TOL	1.E-5
2C:	NBNC NLCZ NPNB	1 1 0
3:	IRAD IUNIT ISTD	0 1 0
4:	TITLE	** 1 MAP UNIT = 1000 FIELD UNITS (FT); TIME IN SECONDS **
4:	SCALE	1000.
5:	Suppress Printout	0 0 0 0 0 0 0
6:	I XG(I) YG(I) H(I) HR(I)	1 1. 1. 100. 0.
6:	I XG(I) YG(I) H(I) HR(I)	2 2. 1. 100. 0.
6:	I XG(I) YG(I) H(I) HR(I)	3 2. 2. 100. 0.
6:	I XG(I) YG(I) H(I) HR(I)	4 1. 2. 100. 0.
7:	I QWEL	1 -.5
9:	J HB	2 100.
9:	J HB	3 100.
10:	Hydraulic Properties	1 1 1E-5 1E-5 0. 0. .1 0.
11:	IEL ND(I), I=1,4	1 1 2 3 4
12A:	IPTK IPTP	0 0
12B:	THK(I)	100. 100. 100. 100.
12C:	TOP(I)	101. 101. 101. 101.
12D:	KZ NO SY	1 1 .1
13A:	IPNC IPNP	0 0
13B:	KZ NO IZIN	1 1 0
13D:	Nonlinear-Boundary Side	1 1 4 .0002 102. 102. 60. 60.
17A:	Stress-Period Indicators	3 0 0 0 0 0 0 0 0
17B:	DELT(I)	600. 600. 600.
17A:	Stress-Period Indicators	3 0 0 0 0 0 0 0 1
17B:	DELT(I)	200. 300. 500.
24A:	N NGNCH	1 0
24B:	J HRK(J) HRL(J)	1 85. 85.
17A:	Stress-Period Indicators	1 0 0 0 0 0 0 0 1
17B:	DELT(I)	200.
24A:	N NGNCH	1 0
24B:	J HRK(J) HRL(J)	1 80. 80.

Table 9.—Data input and descriptions for third example problem

Input Type	Description	Input
1:	TITLE	*** EXAMPLE 3. -- TWO-ELEMENT, FOUR-NODE MESH SIMULATING STEADY-STATE,
1:	TITLE	*** NONLINEAR FLOW; WATER-TABLE CONDITIONS AND POINT AND AREALLY DISTRI-
1:	TITLE	*** BUTED LEAKAGE FUNCTIONS. SOLVED BY COMBINING NONLINEAR SSCG MODELS.
2:	Problem Specifications	2 4 1 1 1 0 0 0 2 4 10
2A:	TOL	.1
2B:	NITSW TOLSW DSMX	10 .0001 10.
2C:	NBNC NLCZ NPNB	0 0 1
2D:	NVNZ	1
3:	IRAD IUNIT ISTD	0 1 1
4:	TITLE	** 1 MAP UNIT = 1000 FIELD UNITS (FT); TIME UNITS IN SECONDS**
4:	SCALE	1000.
5:	Suppress Printout	0 0 0 0 0 0 0
6:	I XG(I) YG(I) H(I) HR(I)	1 1. 1. 100. 100.
6:	I XG(I) YG(I) H(I) HR(I)	2 2. 1. 100. 100.
6:	I XG(I) YG(I) H(I) HR(I)	3 2. 2. 100. 100.
6:	I XG(I) YG(I) H(I) HR(I)	4 1. 2. 100. 100.
9:	J HB	2 100.
9:	J HB	3 100.
10:	Hydraulic Properties	1 2 1E-5 1E-5 0. 0. 0. 0.
11:	IEL ND(I), I=1,4	1 1 2 3 0
11:	IEL ND(I), I=5,8	2 1 3 4 0
12A:	IPTK IPTP	0 0
12B:	THK(I)	100. 100. 100. 100.
12C:	TOP(I)	101. 101. 101. 101.
13A:	IPNC IPNP	0 0
14:	I KP(I) GCP HZP	1 4 .01 89.
15A:	IPNV IPHS	0 0
15B:	L NBE NO VNCF	1 1 2-2.2018E-8
15C:	HS(I)	90. 90. 90. 90.

Table 10.—Data input and descriptions for fourth example problem

Input Type Description	Input										
1: TITLE	MOENCH AND PRICKETT TEST PROBLEM										
1: TITLE	STORAGE CONVERSION										
1: TITLE	**********************************										
2: Problem Specifications	34	52	44	1	1	1	2	1	0	5	20
2A: TOL	.0001										
2C: NBNC NLCZ NPNB	0	0	0								
3: IRAD IUNIT ISTD	0	0	0								
5: Suppress Printout	0	0	0	0	0	0	0				
6: I XG(I) YG(I) H(I) HR(I)	1		0		0		0.		0.		
	2		122.60		-24.386						
. . . . .	3		125		0						
. . . . .	4		122.60		24.386						
. . . . .	5		173.38		-34.488						
	6		176.78		0						
	7		173.38		34.488						
	8		245.20		-48.773						
	9		250		0						
	10		245.20		48.773						
	11		346.76		-68.974						
	12		353.55		0						
	13		346.76		68.974						
	14		490.39		-97.545						
	15		500		0						
	16		490.39		97.545						
	17		693.52		-137.95						
	18		707.11		0						
	19		693.52		137.95						
	20		980.79		-195.09						
	21		1000		0						
	22		980.79		195.09						
	23		1387.04		-275.90						
	24		1414.21		0						
	25		1387.04		275.90						
	26		1961.57		-390.18						
	27		2000		0						
	28		1961.51		390.18						
	29		2774.05		-551.79						
	30		2828.4		0						
	31		2774.05		551.79						
	32		3923.14		-780.36						
	33		4000		0						
	34		3923.14		780.36						
	35		5548.20		-1103.61						
	36		5656.9		0						
	37		5548.20		1103.61						
	38		7846.28		-1560.72						
	39		8000		0						
	40		7846.28		1560.72						
	41		11096.3		-2207.19						
	42		11313.7		0						
	43		11096.3		2207.19						
	44		15692.6		-3121.45						
	45		16000		0						
	46		15692.6		3121.45						
	47		22192.6		-4414.39						
. . . . .	48		22627.4		0						
. . . . .	49		22192.6		4414.39						
. . . . .	50		31385.1		-6242.89						
	51		32000		0						
6: I XG(I) YG(I) H(I) HR(I)	52		31385.1		6242.89		0.		0.		
7: I QWEL	1-2099.4375										

Table 10.—Data input and descriptions for fourth example problem—Continued

Input Type Description	Input								
8A: KZ NOS IZIN	1	2	1						
8B: ALPHZ QBNZ	.04558	0.							
8B: J KQB(J) LQB(J) HK(J) HL(J)	1	50	51	0.	0.				
8B: J KQB(J) LQB(J) HK(J) HL(J)	2	51	52	0.	0.				
10: Hydraulic Properties	1	34	26.73	26.73	0	0	.0001	0	
11: IEL ND(I), I=1,4	1	1	2	3	0				
	2	1	3	4	0				
. . .	3	5	6	3	2				
. . .	4	3	6	7	4				
. . .	5	8	9	6	5				
	6	6	9	10	7				
	7	11	12	9	8				
	8	9	12	13	10				
	9	14	15	12	11				
	10	12	15	16	13				
	11	17	18	15	14				
	12	15	18	19	16				
	13	20	21	18	17				
	14	18	21	22	19				
	15	23	24	21	20				
	16	21	24	25	22				
	17	26	27	24	23				
	18	24	27	28	25				
	19	29	30	27	26				
	20	27	30	31	28				
	21	32	33	30	29				
	22	30	33	34	31				
	23	35	36	33	32				
	24	33	36	37	34				
	25	38	39	36	35				
	26	36	39	40	37				
	27	41	42	39	38				
	28	39	42	43	40				
	29	44	45	42	41				
. . .	30	42	45	46	43				
. . .	31	47	48	45	44				
. . .	32	45	48	49	46				
	33	50	51	48	47				
11: IEL ND(I), I=133,136	34	48	51	52	49				
12A: IPTK IPTP	0	0							
12B: THK(I)	100	100	100	100	100	100	100	100	
	100	100	100	100	100	100	100	100	
.	100	100	100	100	100	100	100	100	
.	100	100	100	100	100	100	100	100	
.	100	100	100	100	100	100	100	100	
	100	100	100	100	100	100	100	100	
12B: THK(I)	100	100	100	100					
12C: TOP(I)	-2	-2	-2	-2	-2	-2	-2	-2	
	-2	-2	-2	-2	-2	-2	-2	-2	
.	-2	-2	-2	-2	-2	-2	-2	-2	
.	-2	-2	-2	-2	-2	-2	-2	-2	
.	-2	-2	-2	-2	-2	-2	-2	-2	
	-2	-2	-2	-2	-2	-2	-2	-2	
12C: TOP(I)	-2	-2	-2	-2					
12D: KZ NO SY	1	34	.1						
13A: IPNC IPNP	0	0							
17A: Stress-Period Indicators	44	0	0	0	0	0	0	0	0
17B: DELT(I)	.00005	.00005	.00005	.00005	.00007	.00011	.00014	.00018	
.	.0003	.0004	.0006	.0007	.0011	.0014	.0018	.003	
.	.004	.006	.007	.011	.014	.018	.03	.04	
.	.06	.07	.11	.14	.18	.3	.4	.6	
	.7	1.1	1.4	1.8	3	4	6	7	
17B: DELT(I)	11	14	18	30					

Examples of Model Output

Output is listed from simulations corresponding to the four examples of input that were given in the previous section. A description of the simulation is given in the first three lines of output for each example. Note that output for the fourth example problem has been edited so that the computed hydraulic head and water-balance summary for the first and last (44th) time steps follow the output of initial conditions.

```
***  EXAMPLE 1. -- TWO-ELEMENT, FOUR-NODE PROBLEM SOLVED WITH LMFE1.
***  CONTAINS THREE STRESS PERIODS WITH CHANGES TO SPECIFIED HEADS
***  AND AREALLY DISTRIBUTED STRESSES ON EACH STRESS PERIOD.

NO. OF ELEMENTS (NELS)............................ =     2
NO. OF NODES (NNDS)............................... =     4
MAX. NO. OF TIME STEPS PER STRESS PERIOD (MXSTPS).. =    1
NO. OF STRESS PERIODS (NPER)...................... =     3
NO. OF ZONES (NZNS)............................... =     1
NO. OF POINT FLOWS (NWELS)........................ =     0
NO. OF CAUCHY-TYPE BOUNDARY ELEMENT SIDES (NQBND).. =    1
NO. OF CAUCHY-TYPE BOUNDARY ZONES (NBCZ).......... =     1
NO. OF SPECIFIED HEADS (NHDS)..................... =     2
MAXIMUM CONDENSED BAND WIDTH (MBWC)............... =     4
MAXIMUM MATRIX BAND WIDTH (MBW)................... =     2

DIMENSION OF G MUST BE AT LEAST     87

SCALE CHANGE FOR NODAL COORDINATES:
** 1 MAP UNIT = 1000 FIELD UNITS (FT); TIME IN SECONDS **

                             NODAL COORDINATES
  NODE      X COORD       Y COORD       NODE     X COORD        Y COORD
    1       1.0000        1.0000          3      2.0000         2.0000
    2       2.0000        1.0000          4      1.0000         2.0000

                              INITIAL HEADS
  NODE           HEAD      NODE        HEAD          NODE         HEAD
    1          100.00        3       100.00
    2          100.00        4       100.00

                             SOURCE BED HEADS
  NODE           HEAD      NODE        HEAD          NODE         HEAD
    1         0.00000        3      0.00000
    2         0.00000        4      0.00000

                 CAUCHY-TYPE BOUNDARY DATA BY BOUNDARY ZONE

     ZONE     1    CONTAINS     1 BOUNDARY SIDES
 SIDE  BOUNDARY  BOUNDARY              SPECIFIED      EXTERNAL      EXTERNAL
  NO.   NODE A    NODE B   ALPHA          FLOW         HEAD A        HEAD B
   1       1         4   0.00000      0.20000E-02  0.00000       0.00000

    SPECIFIED HEADS
             BOUNDARY
  NODE         HEAD
    2        90.000
    3        90.000

                                PARAMETERS BY ZONE
                                ROTATION      AQUITARD      STORAGE     RECHARGE
  ZONE  X TRANS.    Y TRANS.     ANGLE       HYD. COND.  COEFFICIENT     RATE
    1  0.10000E-02 0.10000E-02 0.00000      0.00000     0.10000E-02 0.00000

                 ELEMENT DATA
 ELEMENT  NODE 1  NODE 2  NODE 3  NODE 4   ZONE
    1        1       2       3       0       1
    2        1       3       4       0       1

MATRIX BAND WIDTH (IBND) =   2

                   STRESS PERIOD   1:   TIME STEP SIZES
   NO.       DELTA T          NO.      DELTA T          NO.       DELTA T
    1        5000.0
```

```
          OUTPUT FOR TIME STEP NO.    1 AT  5000.0     TIME UNITS

                         COMPUTED VALUES OF HYDRAULIC HEAD
   NODE         HEAD          NODE         HEAD          NODE          HEAD
     1        114.95            3         90.000
     2        90.000            4         129.46

        SUMMARY OF FLOW AT CAUCHY-TYPE BOUNDARIES BY ZONE

 ZONE     1
 VOLUMETRIC RECHARGE RATE............................ =  2.0000
 VOLUMETRIC DISCHARGE RATE........................... = 0.00000
 TOTAL RECHARGE VOLUME............................... =  10000.
 TOTAL DISCHARGE VOLUME.............................. = 0.00000
 NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... =  2.0000
 NET VOLUME, POSITIVE FOR ACCUMULATION............... =  10000.


     VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE     1

     BOUNDARY     NODE     NODE      VOLUMETRIC FLOW RATES
       SIDE         K        L       NODE K          NODE L
         1          1        4       1.0000          1.0000
```

```
                        WATER BALANCE SUMMARY

             VOLUMETRIC RATES FOR TIME STEP NO.   1
ACCUMULATION OF WATER IN STORAGE.................... = 0.97853
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = 0.00000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. =  2.0000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.00000
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -1.0215
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = 0.23842E-06

           TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... =  4892.7
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = 0.00000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. =  10000.
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.00000
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -5107.3
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = 0.11921E-02

                       STRESS PERIOD    2:    TIME STEP SIZES
   NO.       DELTA T          NO.       DELTA T          NO.       DELTA T
    1        2000.0
```

```
              CHANGES IN VALUES OF DISTRIBUTED FLOWS
BEGINNING ON TIME STEP NO.    1 AT  5000.0     TIME UNITS
               BEG.   NO.   OLD UNIT        NEW UNIT
        ZONE    EL.  ELS.  DISCHARGE       DISCHARGE
          1      1     2   0.00000         0.10000E-05
```

```
                  CHANGES IN VALUES OF SPECIFIED HEAD
BEGINNING ON TIME STEP NO.    1 AT  5000.0     TIME UNITS
                        NODE                HEAD
                          2               110.00
                          3               110.00
```

```
          OUTPUT FOR TIME STEP NO.   1 AT  7000.0     TIME UNITS

                   COMPUTED VALUES OF HYDRAULIC HEAD
    NODE        HEAD        NODE        HEAD        NODE        HEAD
     1         122.95         3        110.00
     2         110.00         4        143.14

         SUMMARY OF FLOW AT CAUCHY-TYPE BOUNDARIES BY ZONE

  ZONE     1
  VOLUMETRIC RECHARGE RATE............................ =  2.0000
  VOLUMETRIC DISCHARGE RATE........................... = 0.00000
  TOTAL RECHARGE VOLUME............................... =  4000.0
  TOTAL DISCHARGE VOLUME.............................. = 0.00000
  NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... =  2.0000
  NET VOLUME, POSITIVE FOR ACCUMULATION............... =  4000.0


      VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE      1

      BOUNDARY     NODE     NODE      VOLUMETRIC FLOW RATES
        SIDE        K        L        NODE K          NODE L
         1          1        4        1.0000          1.0000
```

```
                    WATER BALANCE SUMMARY

          VOLUMETRIC RATES FOR TIME STEP NO.   1
ACCUMULATION OF WATER IN STORAGE.................... =  7.4739
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = 0.00000
RECHARGE FROM DISTRIBUTED SOURCES................... =  1.0000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. =  2.0000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... =  4.4739
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = 0.00000

        TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... =  19840.
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = 0.00000
RECHARGE FROM DISTRIBUTED SOURCES................... =  2000.0
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. =  14000.
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... =  8947.8
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -5107.3
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = 0.11921E-02

                   STRESS PERIOD   3:   TIME STEP SIZES
   NO.      DELTA T        NO.     DELTA T        NO.      DELTA T
    1       3000.0
```

```
               CHANGES IN VALUES OF DISTRIBUTED FLOWS
  BEGINNING ON TIME STEP NO.     1 AT  7000.0     TIME UNITS
                   BEG.   NO.   OLD UNIT      NEW UNIT
           ZONE    EL.   ELS.  DISCHARGE     DISCHARGE
             1      1      2   0.10000E-05   0.00000
```

```
          CHANGES IN VALUES OF SPECIFIED HEAD
BEGINNING ON TIME STEP NO.    1 AT  7000.0     TIME UNITS
                  NODE               HEAD
                   2               100.00
                   3               100.00
```

```
         OUTPUT FOR TIME STEP NO.   1 AT  10000.     TIME UNITS

                      COMPUTED VALUES OF HYDRAULIC HEAD
   NODE          HEAD          NODE         HEAD          NODE          HEAD
    1          131.95            3         100.00
    2          100.00            4         160.45

        SUMMARY OF FLOW AT CAUCHY-TYPE BOUNDARIES BY ZONE

 ZONE      1
 VOLUMETRIC RECHARGE RATE............................ =  2.0000
 VOLUMETRIC DISCHARGE RATE........................... = 0.00000
 TOTAL RECHARGE VOLUME............................... =  6000.0
 TOTAL DISCHARGE VOLUME.............................. = 0.00000
 NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... =  2.0000
 NET VOLUME, POSITIVE FOR ACCUMULATION............... =  6000.0


     VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE      1

     BOUNDARY    NODE    NODE      VOLUMETRIC FLOW RATES
       SIDE        K       L       NODE K          NODE L
        1          1       4       1.0000          1.0000
```

```
                        WATER BALANCE SUMMARY

            VOLUMETRIC RATES FOR TIME STEP NO.   1
ACCUMULATION OF WATER IN STORAGE.................... = 0.29485
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = 0.00000
RECHARGE FROM DISTRIBUTED SOURCES................... = -.11921E-06
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. =  2.0000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.00000
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -1.7051
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = 0.23842E-06

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... =  20725.
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = 0.00000
RECHARGE FROM DISTRIBUTED SOURCES................... =  2000.0
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. =  20000.
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... =  8947.8
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -10223.
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = 0.19073E-02
```

```
*** EXAMPLE 2. -- TWO-ELEMENT, FOUR-NODE PROBLEM SIMULATING WATER-TABLE CONDI-
*** TIONS.  THREE STRESS PERIODS SHOW CHANGES TO CONTROLLING HEADS AT NONLIN-
*** EAR CAUCHY-TYPE BOUNDARIES AND TIME-STEP SIZES.  SOLUTION BY MICCG METHOD.

NO. OF ELEMENTS (NELS)............................ =      1
NO. OF NODES (NNDS)............................... =      4
MAX. NO. OF TIME STEPS PER STRESS PERIOD (MXSTPS).. =     3
NO. OF STRESS PERIODS (NPER)...................... =      3
NO. OF ZONES (NZNS)............................... =      1
NO. OF POINT FLOWS (NWELS)........................ =      1
NO. OF CAUCHY-TYPE BOUNDARY ELEMENT SIDES (NQBND).. =     0
NO. OF CAUCHY-TYPE BOUNDARY ZONES (NBCZ).......... =      0
NO. OF SPECIFIED HEADS (NHDS)..................... =      2
MAXIMUM CONDENSED BAND WIDTH (MBWC)............... =      4
MAXIMUM NO. OF ITERATIONS FOR MICCG (NIT)......... =     10
CLOSURE TOLERANCE FOR MICCG (TOL)................. = 0.10000E-04

DIMENSION OF G MUST BE AT LEAST     71

WATER-TABLE AQUIFER:
NOW G MUST BE DIMENSIONED TO AT LEAST     85

NO. OF NONLINEAR CAUCHY-TYPE BOUNDARIES (NBNC)..... =     1
NO. OF NONLINEAR CAUCHY-TYPE BOUNDARY ZONES (NLCZ). =     1
NO. OF NONLINEAR POINT SINKS (NPNB)............... =      0

NONLINEAR CAUCHY-TYPE BOUNDARIES AND (OR) NONLINEAR POINT SINKS:
NOW G MUST BE DIMENSIONED TO AT LEAST     94

SCALE CHANGE FOR NODAL COORDINATES:
** 1 MAP UNIT = 1000 FIELD UNITS (FT); TIME IN SECONDS **   4: TITLE

                          NODAL COORDINATES
  NODE      X COORD         Y COORD        NODE       X COORD          Y COORD
    1       1.0000          1.0000           3        2.0000           2.0000
    2       2.0000          1.0000           4        1.0000           2.0000

                            INITIAL HEADS
  NODE         HEAD         NODE        HEAD             NODE          HEAD
    1        100.00           3       100.00
    2        100.00           4       100.00

                          SOURCE BED HEADS
  NODE         HEAD         NODE        HEAD             NODE          HEAD
    1        0.00000          3      0.00000
    2        0.00000          4      0.00000

      POINT FLOWS
  NODE      DISCHARGE
    1       -.50000

    SPECIFIED HEADS
                BOUNDARY
  NODE            HEAD
    2           100.00
    3           100.00

                              PARAMETERS BY ZONE
                              ROTATION      AQUITARD      STORAGE      RECHARGE
  ZONE  X TRANS.    Y TRANS.    ANGLE       HYD. COND.  COEFFICIENT      RATE
    1  0.10000E-04 0.10000E-04 0.00000      0.00000      0.10000       0.00000

                     ELEMENT DATA
 ELEMENT   NODE 1   NODE 2   NODE 3   NODE 4    ZONE
     1        1        2        3        4        1
```

```
                         INITIAL AQUIFER THICKNESS
   NODE      THICKNESS      NODE      THICKNESS        NODE      THICKNESS
     1        100.00          3        100.00
     2        100.00          4        100.00

                          ELEVATION OF AQUIFER TOP
   NODE      ELEVATION      NODE      ELEVATION        NODE      ELEVATION
     1        101.00          3        101.00
     2        101.00          4        101.00

   ZONE      NO.     SPECIFIC
            ELS.      YIELD
     1        1     0.10000

             NONLINEAR CAUCHY-TYPE BOUNDARY DATA BY BOUNDARY ZONE

      ZONE      1     CONTAINS      1 BOUNDARY SIDES
        SIDE       LEAKAGE                    EXTERNAL     CONTROLLING
         NO.     COEFFICIENT         NODE        HEAD        ELEVATION
          1       0.20000E-03          1        102.00        60.000
                                       4        102.00        60.000

                     STRESS PERIOD    1:    TIME STEP SIZES
    NO.       DELTA T            NO.      DELTA T             NO.      DELTA T
     1        600.00              2       600.00               3       600.00

 SOLUTION CONVERGED IN     2 ITERATIONS

 SOLUTION CONVERGED IN     1 ITERATIONS

    SUMMARY OF FLOW AT NONLINEAR CAUCHY-TYPE BOUNDARIES BY ZONE

 ZONE     1
 VOLUMETRIC RECHARGE RATE........................... = 0.39988
 VOLUMETRIC DISCHARGE RATE.......................... = 0.00000
 TOTAL RECHARGE VOLUME.............................. =  239.93
 TOTAL DISCHARGE VOLUME............................. = 0.00000
 NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... = 0.39988
 NET VOLUME, POSITIVE FOR ACCUMULATION............... =  239.93

     VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE      1

     BOUNDARY     NODE     NODE      VOLUMETRIC FLOW RATES
       SIDE        K        L       NODE K          NODE L
        1          1        4     0.20036          0.19952
```

```
        OUTPUT FOR TIME STEP NO.   1 AT  600.00     TIME UNITS

                     COMPUTED VALUES OF HYDRAULIC HEAD
   NODE          HEAD           NODE          HEAD           NODE          HEAD
     1          99.995            3          100.00
     2          100.00            4          100.01
```

```
                    WATER BALANCE SUMMARY

            VOLUMETRIC RATES FOR TIME STEP NO.    1
ACCUMULATION OF WATER IN STORAGE.................... = -.10012
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -.50000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.18005E-05
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -.23957E-05
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.39988
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = -.85242E-09

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... = -60.072
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -300.00
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.10803E-02
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -.14374E-02
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... =  239.93
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = -.51145E-06

SOLUTION CONVERGED IN    2 ITERATIONS

SOLUTION CONVERGED IN    1 ITERATIONS

   SUMMARY OF FLOW AT NONLINEAR CAUCHY-TYPE BOUNDARIES BY ZONE

ZONE     1
VOLUMETRIC RECHARGE RATE............................ = 0.39970
VOLUMETRIC DISCHARGE RATE........................... = 0.00000
TOTAL RECHARGE VOLUME............................... =  239.82
TOTAL DISCHARGE VOLUME.............................. = 0.00000
NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... = 0.39970
NET VOLUME, POSITIVE FOR ACCUMULATION............... =  239.82


    VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE      1

    BOUNDARY     NODE     NODE      VOLUMETRIC FLOW RATES
      SIDE         K        L        NODE K          NODE L
        1          1        4      0.20090         0.19880
```

OUTPUT FOR TIME STEP NO. 2 AT 1200.0 TIME UNITS

COMPUTED VALUES OF HYDRAULIC HEAD

NODE	HEAD	NODE	HEAD	NODE	HEAD
1	99.989	3	100.00		
2	100.00	4	100.01		

```
                    WATER BALANCE SUMMARY

            VOLUMETRIC RATES FOR TIME STEP NO.   2
ACCUMULATION OF WATER IN STORAGE.................... = -.10030
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -.50000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.44935E-05
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -.59818E-05
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.39970
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = -.61418E-07

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... = -120.25
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -600.00
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.37764E-02
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -.50265E-02
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... =  479.75
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = -.37362E-04

SOLUTION CONVERGED IN    2 ITERATIONS

SOLUTION CONVERGED IN    1 ITERATIONS

   SUMMARY OF FLOW AT NONLINEAR CAUCHY-TYPE BOUNDARIES BY ZONE

ZONE     1
VOLUMETRIC RECHARGE RATE............................ = 0.39953
VOLUMETRIC DISCHARGE RATE........................... = 0.00000
TOTAL RECHARGE VOLUME............................... =  239.72
TOTAL DISCHARGE VOLUME.............................. = 0.00000
NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... = 0.39953
NET VOLUME, POSITIVE FOR ACCUMULATION............... =  239.72


    VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE     1

    BOUNDARY     NODE     NODE      VOLUMETRIC FLOW RATES
      SIDE        K        L        NODE K         NODE L
       1          1        4      0.20144        0.19809
```

OUTPUT FOR TIME STEP NO. 3 AT 1800.0 TIME UNITS

COMPUTED VALUES OF HYDRAULIC HEAD

NODE	HEAD	NODE	HEAD	NODE	HEAD
1	99.984	3	100.00		
2	100.00	4	100.02		

```
                    WATER BALANCE SUMMARY

            VOLUMETRIC RATES FOR TIME STEP NO.   3
ACCUMULATION OF WATER IN STORAGE.................... = -.10048
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -.50000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.71864E-05
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -.95529E-05
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.39953
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = -.17635E-07

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... = -180.54
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -900.00
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.80882E-02
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -.10758E-01
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... =  719.47
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = -.47943E-04

                   STRESS PERIOD   2:   TIME STEP SIZES
   NO.       DELTA T          NO.       DELTA T          NO.       DELTA T
    1        200.00            2        300.00            3        500.00
```

```
              CHANGES IN VALUES OF EXTERNAL HEAD
            FOR NONLINEAR CAUCHY-TYPE BOUNDARIES
  BEGINNING ON TIME STEP NO.    1 AT  1800.0     TIME UNITS
               SIDE                   EXTERNAL
                NO.       NODE          HEAD
                 1          1         85.000
                            4         85.000

 SOLUTION CONVERGED IN    2 ITERATIONS

 SOLUTION CONVERGED IN    1 ITERATIONS

    SUMMARY OF FLOW AT NONLINEAR CAUCHY-TYPE BOUNDARIES BY ZONE

 ZONE     1
 VOLUMETRIC RECHARGE RATE............................ = 0.00000
 VOLUMETRIC DISCHARGE RATE........................... = -2.9985
 TOTAL RECHARGE VOLUME............................... = 0.00000
 TOTAL DISCHARGE VOLUME.............................. = -599.71
 NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... = -2.9985
 NET VOLUME, POSITIVE FOR ACCUMULATION............... = -599.71


    VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE     1

    BOUNDARY    NODE    NODE     VOLUMETRIC FLOW RATES
      SIDE       K       L        NODE K          NODE L
       1         1       4     -1.4976         -1.5009
```

```
          OUTPUT FOR TIME STEP NO.   1 AT  2000.0     TIME UNITS

                        COMPUTED VALUES OF HYDRAULIC HEAD
    NODE         HEAD          NODE         HEAD          NODE         HEAD
      1        99.972            3        100.00
      2        100.00            4        100.00
```

```
                    WATER BALANCE SUMMARY

            VOLUMETRIC RATES FOR TIME STEP NO.   1
ACCUMULATION OF WATER IN STORAGE..................... = -3.4985
RECHARGE FROM POINT SOURCES.......................... = 0.00000
DISCHARGE FROM POINT SINKS........................... = -.50000
RECHARGE FROM DISTRIBUTED SOURCES.................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS..................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE............ = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES............... = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES............ = 0.12076E-04
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = -.47381E-05
DISCHARGE FROM NONLINEAR POINT SINKS................. = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE............... = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES....... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = -2.9985
FLOW IMBALANCE....................................... = 0.29166E-06

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE..................... = -880.24
RECHARGE FROM POINT SOURCES.......................... = 0.00000
DISCHARGE FROM POINT SINKS........................... = -1000.0
RECHARGE FROM DISTRIBUTED SOURCES.................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS..................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE............ = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES............... = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES............ = 0.10503E-01
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = -.11706E-01
DISCHARGE FROM NONLINEAR POINT SINKS................. = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE............... = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES....... =  719.47
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = -599.71
FLOW IMBALANCE....................................... = 0.10389E-04

SOLUTION CONVERGED IN    2 ITERATIONS

SOLUTION CONVERGED IN    1 ITERATIONS

   SUMMARY OF FLOW AT NONLINEAR CAUCHY-TYPE BOUNDARIES BY ZONE

ZONE     1
VOLUMETRIC RECHARGE RATE............................. = 0.00000
VOLUMETRIC DISCHARGE RATE............................ = -2.9945
TOTAL RECHARGE VOLUME................................ = 0.00000
TOTAL DISCHARGE VOLUME............................... = -898.36
NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE...... = -2.9945
NET VOLUME, POSITIVE FOR ACCUMULATION................ = -898.36


    VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE      1

    BOUNDARY     NODE     NODE     VOLUMETRIC FLOW RATES
      SIDE         K        L       NODE K        NODE L
       1           1        4      -1.4960       -1.4985
```

```
          OUTPUT FOR TIME STEP NO.   2 AT  2300.0     TIME UNITS

                       COMPUTED VALUES OF HYDRAULIC HEAD
 NODE          HEAD          NODE          HEAD          NODE          HEAD
   1         99.954            3         100.00
   2         100.00            4         99.977
```

```
                    WATER BALANCE SUMMARY

           VOLUMETRIC RATES FOR TIME STEP NO.   2
ACCUMULATION OF WATER IN STORAGE.................... = -3.4945
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -.50000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.27316E-04
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = -2.9945
FLOW IMBALANCE...................................... = 0.34039E-06

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... = -1928.6
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -1150.0
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.18698E-01
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -.11706E-01
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... =  719.47
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = -1498.1
FLOW IMBALANCE...................................... = 0.11251E-03

SOLUTION CONVERGED IN    2 ITERATIONS

SOLUTION CONVERGED IN    1 ITERATIONS

   SUMMARY OF FLOW AT NONLINEAR CAUCHY-TYPE BOUNDARIES BY ZONE

ZONE     1
VOLUMETRIC RECHARGE RATE............................ = 0.00000
VOLUMETRIC DISCHARGE RATE........................... = -2.9881
TOTAL RECHARGE VOLUME............................... = 0.00000
TOTAL DISCHARGE VOLUME.............................. = -1494.0
NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... = -2.9881
NET VOLUME, POSITIVE FOR ACCUMULATION............... = -1494.0


    VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE      1

    BOUNDARY     NODE     NODE     VOLUMETRIC FLOW RATES
      SIDE         K        L       NODE K         NODE L
        1          1        4     -1.4934        -1.4947
```

```
          OUTPUT FOR TIME STEP NO.   3 AT  2800.0     TIME UNITS

                       COMPUTED VALUES OF HYDRAULIC HEAD
NODE         HEAD         NODE         HEAD         NODE         HEAD
  1        99.924           3        100.00
  2        100.00           4        99.932
```

```
                    WATER BALANCE SUMMARY

            VOLUMETRIC RATES FOR TIME STEP NO.   3
ACCUMULATION OF WATER IN STORAGE.................... = -3.4880
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -.50000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.59697E-04
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = -2.9881
FLOW IMBALANCE...................................... = -.92194E-07

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... = -3672.6
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -1400.0
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.48547E-01
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -.11706E-01
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... =  719.47
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = -2992.1
FLOW IMBALANCE...................................... = 0.66410E-04

                   STRESS PERIOD   3:   TIME STEP SIZES
   NO.      DELTA T          NO.      DELTA T          NO.      DELTA T
    1       200.00
```

```
              CHANGES IN VALUES OF EXTERNAL HEAD
            FOR NONLINEAR CAUCHY-TYPE BOUNDARIES
   BEGINNING ON TIME STEP NO.    1 AT  2800.0     TIME UNITS
               SIDE                    EXTERNAL
                NO.        NODE          HEAD
                 1           1         80.000
                             4         80.000

 SOLUTION CONVERGED IN    2 ITERATIONS

 SOLUTION CONVERGED IN    1 ITERATIONS

    SUMMARY OF FLOW AT NONLINEAR CAUCHY-TYPE BOUNDARIES BY ZONE

 ZONE     1
 VOLUMETRIC RECHARGE RATE........................... = 0.00000
 VOLUMETRIC DISCHARGE RATE.......................... = -3.9830
 TOTAL RECHARGE VOLUME.............................. = 0.00000
 TOTAL DISCHARGE VOLUME............................. = -796.60
 NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE.... = -3.9830
 NET VOLUME, POSITIVE FOR ACCUMULATION.............. = -796.60

     VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE     1

     BOUNDARY     NODE     NODE      VOLUMETRIC FLOW RATES
       SIDE         K        L      NODE K          NODE L
         1          1        4     -1.9914         -1.9916
```

```
         OUTPUT FOR TIME STEP NO.    1 AT  3000.0     TIME UNITS

                   COMPUTED VALUES OF HYDRAULIC HEAD
  NODE         HEAD          NODE         HEAD          NODE         HEAD
   1          99.909          3          100.00
   2          100.00          4          99.908
```

```
                    WATER BALANCE SUMMARY

            VOLUMETRIC RATES FOR TIME STEP NO.   1
ACCUMULATION OF WATER IN STORAGE.................... = -4.4829
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -.50000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.85085E-04
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = -3.9830
FLOW IMBALANCE...................................... = -.20780E-06

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... = -4569.2
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -1500.0
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.65564E-01
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = -.11706E-01
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... =  719.47
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = -3788.7
FLOW IMBALANCE...................................... = 0.24849E-04
```

```
*** EXAMPLE 3. -- TWO-ELEMENT, FOUR-NODE MESH SIMULATING STEADY-STATE,
*** NONLINEAR FLOW; WATER-TABLE CONDITIONS AND POINT AND AREALLY DISTRI-
*** BUTED LEAKAGE FUNCTIONS. SOLVED BY COMBINING NONLINEAR SSCG MODELS.

NO. OF ELEMENTS (NELS)............................. =      2
NO. OF NODES (NNDS)................................ =      4
MAX. NO. OF TIME STEPS PER STRESS PERIOD (MXSTPS).. =      1
NO. OF STRESS PERIODS (NPER)....................... =      1
NO. OF ZONES (NZNS)................................ =      1
NO. OF POINT FLOWS (NWELS)......................... =      0
NO. OF CAUCHY-TYPE BOUNDARY ELEMENT SIDES (NQBND).. =      0
NO. OF CAUCHY-TYPE BOUNDARY ZONES (NBCZ)........... =      0
NO. OF SPECIFIED HEADS (NHDS)...................... =      2
MAXIMUM CONDENSED BAND WIDTH (MBWC)................ =      4
MAXIMUM NO. OF ITERATIONS FOR MICCG (NIT).......... =     10
CLOSURE TOLERANCE FOR MICCG (TOL).................. = 0.10000

DIMENSION OF G MUST BE AT LEAST     75

MAX. NO. OF WATER-TABLE ITERATIONS (NITSW) ........ =     10
CLOSURE TOL. FOR WATER-TABLE ITERATIONS (TOLSW) ... = 0.10000E-03
MAXIMUM ALLOWABLE DISPLACEMENT (DSMX) ............. =  10.000

STEADY-STATE FLOW IN A WATER-TABLE AQUIFER:
NOW G MUST BE DIMENSIONED TO AT LEAST     83

NO. OF NONLINEAR CAUCHY-TYPE BOUNDARIES (NBNC)..... =      0
NO. OF NONLINEAR CAUCHY-TYPE BOUNDARY ZONES (NLCZ). =      0
NO. OF NONLINEAR POINT SINKS (NPNB)................ =      1

NONLINEAR CAUCHY-TYPE BOUNDARIES AND (OR) NONLINEAR POINT SINKS:
NOW G MUST BE DIMENSIONED TO AT LEAST     86

NO. OF NONLINEAR STEADY-LEAKAGE ZONES (NVNZ)....... =      1

NONLINEAR STEADY LEAKAGE:
NOW G MUST BE DIMENSIONED TO AT LEAST     94

** STEADY-STATE FLOW **

SCALE CHANGE FOR NODAL COORDINATES:
** 1 MAP UNIT = 1000 FIELD UNITS (FT); TIME UNITS IN SECONDS** 4: TITLE

                            NODAL COORDINATES
  NODE      X COORD       Y COORD       NODE        X COORD        Y COORD
   1        1.0000        1.0000          3         2.0000         2.0000
   2        2.0000        1.0000          4         1.0000         2.0000

                             INITIAL HEADS
  NODE          HEAD       NODE       HEAD         NODE            HEAD
   1          100.00         3      100.00
   2          100.00         4      100.00

                           SOURCE BED HEADS
  NODE          HEAD       NODE       HEAD         NODE            HEAD
   1          100.00         3      100.00
   2          100.00         4      100.00

    SPECIFIED HEADS
             BOUNDARY
  NODE          HEAD
   2          100.00
   3          100.00
```

PARAMETERS BY ZONE

ZONE	X TRANS.	Y TRANS.	ROTATION ANGLE	AQUITARD HYD. COND.	STORAGE COEFFICIENT	RECHARGE RATE
1	0.10000E-04	0.10000E-04	0.00000	0.00000	0.00000	0.00000

ELEMENT DATA

ELEMENT	NODE 1	NODE 2	NODE 3	NODE 4	ZONE
1	1	2	3	0	1
2	1	3	4	0	1

INITIAL AQUIFER THICKNESS

NODE	THICKNESS	NODE	THICKNESS	NODE	THICKNESS
1	100.00	3	100.00		
2	100.00	4	100.00		

ELEVATION OF AQUIFER TOP

NODE	ELEVATION	NODE	ELEVATION	NODE	ELEVATION
1	101.00	3	101.00		
2	101.00	4	101.00		

NONLINEAR POINT SINK DATA

POINT NO.	NODE NO.	LEAKAGE COEFFICIENT	CONTROLLING ELEVATION
1	4	0.10000E-01	89.000

PARAMETERS FOR
NONLINEAR STEADY LEAKAGE BY ZONE

ZONE	FIRST EL. NO.	NO. OF ELEMENTS	COEFFICIENT VALUE
1	1	2	-.22018E-07

CONTROL ELEVATION FOR NONLINEAR STEADY LEAKAGE

NODE	VALUE	NODE	VALUE	NODE	VALUE
1	90.000	3	90.000		
2	90.000	4	90.000		

SOLUTION CONVERGED IN 2 ITERATIONS

SOLUTION CONVERGED IN 2 ITERATIONS

SOLUTION CONVERGED IN 2 ITERATIONS

SOLUTION CONVERGED IN 1 ITERATIONS

SOLUTION CONVERGED IN 1 ITERATIONS

```
                  SUMMARY OF CLOSURE INFORMATION
NO. OF ITERATIONS TO CLOSE (ITER) .................. =     5
MAXIMUM ABSOLUTE DISPLACEMENT (DSPA) ............... = 0.58929E-05
```

```
          OUTPUT FOR TIME STEP NO.   1 AT  1.0000     TIME UNITS

                         COMPUTED VALUES OF HYDRAULIC HEAD
  NODE        HEAD           NODE        HEAD           NODE          HEAD
    1        90.551            3        100.00
    2        100.00            4        89.541

                                SATURATED THICKNESS
  NODE     THICKNESS         NODE     THICKNESS         NODE      THICKNESS
    1        90.551            3        100.00
    2        100.00            4        89.541
```

```
                       WATER BALANCE SUMMARY

            VOLUMETRIC RATES FOR TIME STEP NO.    1
ACCUMULATION OF WATER IN STORAGE.................... = 0.00000
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = 0.00000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.11955
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................ = -.54108E-02
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = -.11414
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = -.89407E-07

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... = 0.00000
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = 0.00000
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.11955
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................ = -.54108E-02
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = -.11414
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = -.89407E-07
```

```
MOENCH AND PRICKETT TEST PROBLEM
STORAGE CONVERSION
*********************************

NO. OF ELEMENTS (NELS)............................ =    34
NO. OF NODES (NNDS)............................... =    52
MAX. NO. OF TIME STEPS PER STRESS PERIOD (MXSTPS).. =    44
NO. OF STRESS PERIODS (NPER)...................... =     1
NO. OF ZONES (NZNS)............................... =     1
NO. OF POINT FLOWS (NWELS)........................ =     1
NO. OF CAUCHY-TYPE BOUNDARY ELEMENT SIDES (NQBND).. =     2
NO. OF CAUCHY-TYPE BOUNDARY ZONES (NBCZ).......... =     1
NO. OF SPECIFIED HEADS (NHDS)..................... =     0
MAXIMUM CONDENSED BAND WIDTH (MBWC)............... =     5
MAXIMUM NO. OF ITERATIONS FOR MICCG (NIT)......... =    20
CLOSURE TOLERANCE FOR MICCG (TOL)................. = 0.10000E-03

DIMENSION OF G MUST BE AT LEAST   1260

WATER-TABLE AQUIFER:
NOW G MUST BE DIMENSIONED TO AT LEAST   1468

NO. OF NONLINEAR CAUCHY-TYPE BOUNDARIES (NBNC)..... =     0
NO. OF NONLINEAR CAUCHY-TYPE BOUNDARY ZONES (NLCZ). =     0
NO. OF NONLINEAR POINT SINKS (NPNB)................ =     0

NONLINEAR CAUCHY-TYPE BOUNDARIES AND (OR) NONLINEAR POINT SINKS:
NOW G MUST BE DIMENSIONED TO AT LEAST   1468
```

NODAL COORDINATES

NODE	X COORD	Y COORD	NODE	X COORD	Y COORD
1	0.00000	0.00000	27	2000.0	0.00000
2	122.60	-24.386	28	1961.5	390.18
3	125.00	0.00000	29	2774.0	-551.79
4	122.60	24.386	30	2828.4	0.00000
5	173.38	-34.488	31	2774.0	551.79
6	176.78	0.00000	32	3923.1	-780.36
7	173.38	34.488	33	4000.0	0.00000
8	245.20	-48.773	34	3923.1	780.36
9	250.00	0.00000	35	5548.2	-1103.6
10	245.20	48.773	36	5656.9	0.00000
11	346.76	-68.974	37	5548.2	1103.6
12	353.55	0.00000	38	7846.3	-1560.7
13	346.76	68.974	39	8000.0	0.00000
14	490.39	-97.545	40	7846.3	1560.7
15	500.00	0.00000	41	11096.	-2207.2
16	490.39	97.545	42	11314.	0.00000
17	693.52	-137.95	43	11096.	2207.2
18	707.11	0.00000	44	15693.	-3121.5
19	693.52	137.95	45	16000.	0.00000
20	980.79	-195.09	46	15693.	3121.5
21	1000.0	0.00000	47	22193.	-4414.4
22	980.79	195.09	48	22627.	0.00000
23	1387.0	-275.90	49	22193.	4414.4
24	1414.2	0.00000	50	31385.	-6242.9
25	1387.0	275.90	51	32000.	0.00000
26	1961.6	-390.18	52	31385.	6242.9

INITIAL HEADS

NODE	HEAD	NODE	HEAD	NODE	HEAD
1	0.00000	19	0.00000	37	0.00000
2	0.00000	20	0.00000	38	0.00000
3	0.00000	21	0.00000	39	0.00000
4	0.00000	22	0.00000	40	0.00000
5	0.00000	23	0.00000	41	0.00000

6	0.00000	24	0.00000	42	0.00000
7	0.00000	25	0.00000	43	0.00000
8	0.00000	26	0.00000	44	0.00000
9	0.00000	27	0.00000	45	0.00000
10	0.00000	28	0.00000	46	0.00000
11	0.00000	29	0.00000	47	0.00000
12	0.00000	30	0.00000	48	0.00000
13	0.00000	31	0.00000	49	0.00000
14	0.00000	32	0.00000	50	0.00000
15	0.00000	33	0.00000	51	0.00000
16	0.00000	34	0.00000	52	0.00000
17	0.00000	35	0.00000		
18	0.00000	36	0.00000		

SOURCE BED HEADS

NODE	HEAD	NODE	HEAD	NODE	HEAD
1	0.00000	19	0.00000	37	0.00000
2	0.00000	20	0.00000	38	0.00000
3	0.00000	21	0.00000	39	0.00000
4	0.00000	22	0.00000	40	0.00000
5	0.00000	23	0.00000	41	0.00000
6	0.00000	24	0.00000	42	0.00000
7	0.00000	25	0.00000	43	0.00000
8	0.00000	26	0.00000	44	0.00000
9	0.00000	27	0.00000	45	0.00000
10	0.00000	28	0.00000	46	0.00000
11	0.00000	29	0.00000	47	0.00000
12	0.00000	30	0.00000	48	0.00000
13	0.00000	31	0.00000	49	0.00000
14	0.00000	32	0.00000	50	0.00000
15	0.00000	33	0.00000	51	0.00000
16	0.00000	34	0.00000	52	0.00000
17	0.00000	35	0.00000		
18	0.00000	36	0.00000		

POINT FLOWS

NODE	DISCHARGE
1	-2099.4

CAUCHY-TYPE BOUNDARY DATA BY BOUNDARY ZONE

ZONE 1 CONTAINS 2 BOUNDARY SIDES

SIDE NO.	BOUNDARY NODE A	BOUNDARY NODE B	ALPHA	SPECIFIED FLOW	EXTERNAL HEAD A	EXTERNAL HEAD B
1	50	51	0.45580E-01	0.00000	0.00000	0.00000
2	51	52	0.45580E-01	0.00000	0.00000	0.00000

PARAMETERS BY ZONE

ZONE	X TRANS.	Y TRANS.	ROTATION ANGLE	AQUITARD HYD. COND.	STORAGE COEFFICIENT	RECHARGE RATE
1	26.730	26.730	0.00000	0.00000	0.10000E-03	0.00000

ELEMENT DATA

ELEMENT	NODE 1	NODE 2	NODE 3	NODE 4	ZONE
1	1	2	3	0	1
2	1	3	4	0	1
3	5	6	3	2	1
4	3	6	7	4	1
5	8	9	6	5	1
6	6	9	10	7	1
7	11	12	9	8	1
8	9	12	13	10	1
9	14	15	12	11	1
10	12	15	16	13	1
11	17	18	15	14	1
12	15	18	19	16	1

13	20	21	18	17	1
14	18	21	22	19	1
15	23	24	21	20	1
16	21	24	25	22	1
17	26	27	24	23	1
18	24	27	28	25	1
19	29	30	27	26	1
20	27	30	31	28	1
21	32	33	30	29	1
22	30	33	34	31	1
23	35	36	33	32	1
24	33	36	37	34	1
25	38	39	36	35	1
26	36	39	40	37	1
27	41	42	39	38	1
28	39	42	43	40	1
29	44	45	42	41	1
30	42	45	46	43	1
31	47	48	45	44	1
32	45	48	49	46	1
33	50	51	48	47	1
34	48	51	52	49	1

INITIAL AQUIFER THICKNESS

NODE	THICKNESS	NODE	THICKNESS	NODE	THICKNESS
1	100.00	19	100.00	37	100.00
2	100.00	20	100.00	38	100.00
3	100.00	21	100.00	39	100.00
4	100.00	22	100.00	40	100.00
5	100.00	23	100.00	41	100.00
6	100.00	24	100.00	42	100.00
7	100.00	25	100.00	43	100.00
8	100.00	26	100.00	44	100.00
9	100.00	27	100.00	45	100.00
10	100.00	28	100.00	46	100.00
11	100.00	29	100.00	47	100.00
12	100.00	30	100.00	48	100.00
13	100.00	31	100.00	49	100.00
14	100.00	32	100.00	50	100.00
15	100.00	33	100.00	51	100.00
16	100.00	34	100.00	52	100.00
17	100.00	35	100.00		
18	100.00	36	100.00		

ELEVATION OF AQUIFER TOP

NODE	ELEVATION	NODE	ELEVATION	NODE	ELEVATION
1	-2.0000	19	-2.0000	37	-2.0000
2	-2.0000	20	-2.0000	38	-2.0000
3	-2.0000	21	-2.0000	39	-2.0000
4	-2.0000	22	-2.0000	40	-2.0000
5	-2.0000	23	-2.0000	41	-2.0000
6	-2.0000	24	-2.0000	42	-2.0000
7	-2.0000	25	-2.0000	43	-2.0000
8	-2.0000	26	-2.0000	44	-2.0000
9	-2.0000	27	-2.0000	45	-2.0000
10	-2.0000	28	-2.0000	46	-2.0000
11	-2.0000	29	-2.0000	47	-2.0000
12	-2.0000	30	-2.0000	48	-2.0000
13	-2.0000	31	-2.0000	49	-2.0000
14	-2.0000	32	-2.0000	50	-2.0000
15	-2.0000	33	-2.0000	51	-2.0000
16	-2.0000	34	-2.0000	52	-2.0000
17	-2.0000	35	-2.0000		
18	-2.0000	36	-2.0000		

ZONE NO. SPECIFIC

```
        ELS.      YIELD
   1     34     0.10000

                STRESS PERIOD    1:   TIME STEP SIZES
  NO.       DELTA T        NO.        DELTA T          NO.         DELTA T
   1      0.50000E-04      16       0.30000E-02        31        0.40000
   2      0.50000E-04      17       0.40000E-02        32        0.60000
   3      0.50000E-04      18       0.60000E-02        33        0.70000
   4      0.50000E-04      19       0.70000E-02        34         1.1000
   5      0.70000E-04      20       0.11000E-01        35         1.4000
   6      0.11000E-03      21       0.14000E-01        36         1.8000
   7      0.14000E-03      22       0.18000E-01        37         3.0000
   8      0.18000E-03      23       0.30000E-01        38         4.0000
   9      0.30000E-03      24       0.40000E-01        39         6.0000
  10      0.40000E-03      25       0.60000E-01        40         7.0000
  11      0.60000E-03      26       0.70000E-01        41         11.000
  12      0.70000E-03      27       0.11000            42         14.000
  13      0.11000E-02      28       0.14000            43         18.000
  14      0.14000E-02      29       0.18000            44         30.000
  15      0.18000E-02      30       0.30000

SOLUTION CONVERGED IN    3 ITERATIONS

SOLUTION CONVERGED IN    1 ITERATIONS

        SUMMARY OF FLOW AT CAUCHY-TYPE BOUNDARIES BY ZONE

ZONE     1
VOLUMETRIC RECHARGE RATE............................ = 0.00000
VOLUMETRIC DISCHARGE RATE........................... = 0.00000
TOTAL RECHARGE VOLUME............................... = 0.00000
TOTAL DISCHARGE VOLUME.............................. = 0.00000
NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... = 0.00000
NET VOLUME, POSITIVE FOR ACCUMULATION............... = 0.00000


    VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE      1

    BOUNDARY     NODE     NODE      VOLUMETRIC FLOW RATES
      SIDE        K        L         NODE K         NODE L
       1         50       51       0.00000        0.00000
       2         51       52       0.00000        0.00000
```

OUTPUT FOR TIME STEP NO. 1 AT 0.50000E-04 TIME UNITS

COMPUTED VALUES OF HYDRAULIC HEAD

NODE	HEAD	NODE	HEAD	NODE	HEAD
1	-.88604	19	-.14875E-07	37	-.16577E-24
2	-.35296E-01	20	-.12188E-09	38	-.22262E-28
3	-.33935E-01	21	-.48501E-10	39	-.13388E-29
4	-.35296E-01	22	-.12189E-09	40	-.22261E-28
5	-.57912E-02	23	-.51023E-12	41	-.14955E-32
6	-.52763E-02	24	-.15344E-12	42	-.64049E-34
7	-.57912E-02	25	-.51029E-12	43	-.14954E-32
8	-.56739E-03	26	-.10814E-14	44	-.50003E-37
9	-.47791E-03	27	-.24065E-15	45	0.00000
10	-.56739E-03	28	-.10817E-14	46	-.48803E-37
11	-.31360E-04	29	-.11540E-17	47	0.00000
12	-.23442E-04	30	-.18734E-18	48	0.00000
13	-.31360E-04	31	-.11539E-17	49	0.00000
14	-.94000E-06	32	-.61793E-21	50	0.00000
15	-.59464E-06	33	-.72477E-22	51	0.00000
16	-.94003E-06	34	-.61791E-21	52	0.00000
17	-.14874E-07	35	-.16578E-24		
18	-.76070E-08	36	-.13955E-25		

```
                      WATER BALANCE SUMMARY

             VOLUMETRIC RATES FOR TIME STEP NO.   1
ACCUMULATION OF WATER IN STORAGE.................... = -2099.4
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -2099.4
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.00000
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = 0.00000

           TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE.................... = -.10497
RECHARGE FROM POINT SOURCES......................... = 0.00000
DISCHARGE FROM POINT SINKS.......................... = -.10497
RECHARGE FROM DISTRIBUTED SOURCES................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS.................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE.......... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.00000
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES.......... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................ = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES..... = 0.00000
FLOW IMBALANCE...................................... = 0.00000

SOLUTION CONVERGED IN    7 ITERATIONS

SOLUTION CONVERGED IN    5 ITERATIONS

         SUMMARY OF FLOW AT CAUCHY-TYPE BOUNDARIES BY ZONE

ZONE     1
VOLUMETRIC RECHARGE RATE............................ =  502.60
VOLUMETRIC DISCHARGE RATE........................... = 0.00000
TOTAL RECHARGE VOLUME............................... =  15078.
TOTAL DISCHARGE VOLUME.............................. = 0.00000
NET VOLUMETRIC FLOW RATE, POSITIVE FOR RECHARGE..... =  502.60
NET VOLUME, POSITIVE FOR ACCUMULATION............... =  15078.


    VOLUMETRIC FLOW RATES BY BOUNDARY SIDE FOR ZONE      1

    BOUNDARY    NODE     NODE      VOLUMETRIC FLOW RATES
      SIDE        K        L       NODE K          NODE L
       1         50       51       125.63          125.68
       2         51       52       125.68          125.63
```

OUTPUT FOR TIME STEP NO. 44 AT 100.00 TIME UNITS

COMPUTED VALUES OF HYDRAULIC HEAD

NODE	HEAD	NODE	HEAD	NODE	HEAD
1	-12.166	19	-4.3208	37	-1.8042
2	-7.8350	20	-3.6728	38	-1.6194
3	-7.8350	21	-3.6716	39	-1.6193
4	-7.8350	22	-3.6728	40	-1.6194
5	-7.1132	23	-3.0745	41	-1.4358
6	-7.1131	24	-3.0722	42	-1.4357
7	-7.1132	25	-3.0745	43	-1.4358
8	-6.3984	26	-2.5599	44	-1.2544
9	-6.3983	27	-2.5564	45	-1.2543
10	-6.3984	28	-2.5600	46	-1.2544
11	-5.6923	29	-2.1788	47	-1.0765
12	-5.6921	30	-2.1739	48	-1.0764
13	-5.6923	31	-2.1788	49	-1.0765
14	-4.9978	32	-1.9901	50	-.90430
15	-4.9974	33	-1.9898	51	-.90458
16	-4.9978	34	-1.9901	52	-.90430
17	-4.3208	35	-1.8042		
18	-4.3201	36	-1.8042		

```
                    WATER BALANCE SUMMARY

            VOLUMETRIC RATES FOR TIME STEP NO.  44
ACCUMULATION OF WATER IN STORAGE..................... = -1596.8
RECHARGE FROM POINT SOURCES.......................... = 0.00000
DISCHARGE FROM POINT SINKS........................... = -2099.4
RECHARGE FROM DISTRIBUTED SOURCES.................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS..................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE............ = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES............... =  502.60
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES............ = 0.00000
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................. = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE............... = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES....... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
FLOW IMBALANCE....................................... = 0.24414E-03

          TOTAL VOLUMES SINCE BEGINNING OF SIMULATION
ACCUMULATION OF WATER IN STORAGE..................... = -.17353E+06
RECHARGE FROM POINT SOURCES.......................... = 0.00000
DISCHARGE FROM POINT SINKS........................... = -.20994E+06
RECHARGE FROM DISTRIBUTED SOURCES.................... = 0.00000
DISCHARGE FROM DISTRIBUTED SINKS..................... = 0.00000
RECHARGE FROM STEADY OR TRANSIENT LEAKAGE............ = 0.00000
DISCHARGE FROM STEADY OR TRANSIENT LEAKAGE........... = 0.00000
RECHARGE ACROSS CAUCHY-TYPE BOUNDARIES............... =  36418.
DISCHARGE ACROSS CAUCHY-TYPE BOUNDARIES.............. = 0.00000
RECHARGE ACROSS SPECIFIED-HEAD BOUNDARIES............ = 0.00000
DISCHARGE ACROSS SPECIFIED-HEAD BOUNDARIES........... = 0.00000
DISCHARGE FROM NONLINEAR POINT SINKS................. = 0.00000
RECHARGE FROM NONLINEAR STEADY LEAKAGE............... = 0.00000
DISCHARGE FROM NONLINEAR STEADY LEAKAGE.............. = 0.00000
RECHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES....... = 0.00000
DISCHARGE FROM NONLINEAR CAUCHY-TYPE BOUNDARIES...... = 0.00000
FLOW IMBALANCE....................................... = -.37894E-02
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*U.S. G.P.O.:1993-342-389:80007